Contact Phase Invariant Control for Humanoid Robot based on Variable Impedant Inverted Pendulum Model

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Abstract

As expected for future utilities in the human environment, it is required for humanoid robots to have much higher activity. Realization of transition between contact and aerial phase at will is essential to expand the range of action and variety of motions for them. Manipulation of both the contact condition and the external force is the key to enhance the humanoid mobility since they are driven by the external force, converting the inner force through interaction with the environment. The difficulty lies on the complexity of the dynamics of them that consists of a number of degrees of freedom and varies in accordance with contact phase, which is also a problem in terms of implementation. We propose the Variable Inpedant Inverted Pendulum(VIIP) model control to realize the scheme. The advantage of the proposed is that it is invariant on contact phase so that both motions in contact and in aerial are achieved in the unified way. It also helps the problem of implementation, reducing the amount of computation. We verified the validity of the controller in a computer simulation, using a small humanoid robot model.

1 Introduction

Humanoid robots are expected to be used as superior assistants of human beings in the future, and therefore, they should be given ability to act in the human infrastructures much more freely. For the aim, realization of transition from/to contact phase to/from aerial phase (jumping, for example) at will is an essential motion to expand the range of action and variety of motions for them, although research in this field has mainly focused on walking only in contact phase [1, 2, 3, 4, 5] . It is required in the case that the robot suffers so large disturbance that it is hard to absorb with both feet keeping contact to the ground, or the case that feasible places to land are at a distance from each other. And not to mention, it helps robots to move faster, widening steps.

Some previous researches have achieved jumping and running, typical motions through aerial phase. Raibert et al.[6] realized hopping motion and even somersault with simple body robots by simple controlling method, and thus, it is hard to be applied for humanoid robots due to the complexity of the dynamics of them. Nagasaka[7], Yamane et al.[8] and Kajita et al.[9] developed jumping and running pattern generation methods for humanoids. Motion planning and control, however, are unseparable in nature since humanoid is dominated by non-holonomic constraints. Hirano et al.[10] realized a continuous jumping motion in computer simulation using an adaptive impedance control, which is yet to be augmented to various types of motions through aerial phase. Mita et al.[11] proposed a Variable Constraint Control. Though it is effective against non-holonomic constraint, they treated a strict equation of motion of the robot, and thus, the more number of links it has, the more amount of computation is required. For quick responsive motions, and the amount of computation should be lessened so that controlling period should be short. Arikawa et al.[12] developed a Multi-DOF Jumping Robot, which is controlled according to pre-planned polynomial trajectory, and thus, is not robust for disturbance. Pfeiffer et al.[13] developed Jogging JOHNNIE, aiming at a fast movement through running. It is still under realization.

Since humanoids are driven by the external reaction force which are converted from the inner force(joint torque) through interaction with the environment, manipulation of both the contact condition between the robot and the environment and the external force is the key to spread out the potential of their mobility. And the motion with a transition from/to contact and aerial phase especially requires skillful manipulation of external force. The difficulty lies on the complexity of the dynamics of them that consists of a number of degrees of freedom and varies in accordance with contact phase, which is also a problem in terms of implementation, requiring a large amount of computation in general.

The authors[14] had proposed the controlling method of humanoid robots through indirect manipulation of ZMP [15] based on the similarity between the dynamics of them and that of inverted pendulum which is supported at

ZMP. And using it, responsive motions and robust absorption of unpredicted impacts in contact phase are realized. In this paper, we augment it to realize the transition from/to contact to/from aerial phase. It is shown that such transition can be made easier by introducing the Variable Impedant Inverted Pendulum(VIIP) model. And we propose a new controlling method of humanoid robots based on the model.

2 Variable Inpedant Inverted Pendulum(VIIP) Model

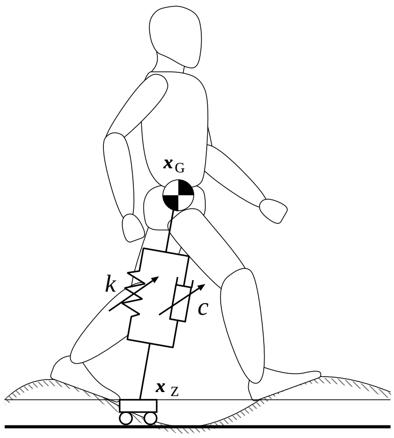


Figure 1: Legged system and variable impedant inverted pendulum

Suppose z-axis coincides with vertical direction, $\boldsymbol{n}_Z$ is the total moment around ZMP, and $\boldsymbol{f}_G = [\, f_x \quad f_y \quad f_z \,]^T$, $\boldsymbol{n}_G = [\, n_x \quad n_y \quad n_z \,]^T$ are the force and moment acting at the center of gravity(COG). In accordance with both the equation of motion and the equilibrium of moment, we get the following equations.

$$m(\ddot{\boldsymbol{x}}_G + \boldsymbol{g}) = \boldsymbol{f}_G \tag{1}$$

$$(\boldsymbol{x}_G - \boldsymbol{x}_Z) \times \boldsymbol{f}_G + \boldsymbol{n}_G = \boldsymbol{n}_Z \tag{2}$$

where $\boldsymbol{g} = [\, 0 \quad 0 \quad g \,]^T$ is the acceleration of gravity. Since both the first and the second component of $\boldsymbol{n}_Z$ are zero by definition of ZMP,

$$\ddot{x}_G = \omega_G^2(x_G - x_Z) - \frac{n_y}{m(z_G - z_Z)} \tag{3}$$

$$\ddot{y}_G = \omega_G^2(y_G - y_Z) + \frac{n_x}{m(z_G - z_Z)} \tag{4}$$

$$\ddot{z}_G = \frac{f_z}{m} - g \tag{5}$$

where $\boldsymbol{x}_G = [\, x_G \quad y_G \quad z_G \,]^T$, $\boldsymbol{x}_Z = [\, x_Z \quad y_Z \quad z_Z \,]^T$ are the position of COG, ZMP respectively, f_z is a vertical ground reaction force, m is a total mass of the robot and ω_G is defined as:

$$\omega_G^2 = \frac{f_z}{m(z_G - z_Z)} \tag{6}$$

Though $\boldsymbol{n}_G$ varies under the influence of the whole body motion, the change of it is less than that of $\boldsymbol{f}_G$ in general. Thus, $\boldsymbol{n}_G$ can be treated as constant values during a short term. And from the equation (3)(4)(5), one can see that COG $\boldsymbol{x}_G$ can be controlled through manipulation of ZMP $\boldsymbol{x}_Z$ and vertical reaction force f_z. Especially, both the equation (3) and (4) shows that the dynamics of humanoid is similar to that of inverted pendulum although it has an offset caused by the existence of $\boldsymbol{n}_G$. Therefore, as we've already shown in [14], decision of the referential ZMP can be modeled upon the controlling strategy of inverted pendulum.

The equation (5), which is concerned with the COG motion along the vertical direction, must satisfy the following constraint.

$$f_z \geq 0 \tag{7}$$

When f_z equals to zero, the humanoid is in aerial phase. The manipulation of f_z plays an important role for the seamless transition from/to contact phase to/from aerial phase. COG should be given an enough velocity and acceleration to detach off the ground in order to shift from contact phase to aerial phase, and from aerial phase to contact phase, the impact at the moment of touchdown should be absorbed. We propose the unified method of such COG acceleration and impact absorption based on the Variable Impedant Inverted Pendulum model control shown in Fig.1.

${}^{ref}f_z$ is decided as:

$${}^{ref}f_z = K_{Pz}({}^{ref}z_G - z_G) + K_{Dz}({}^{ref}\dot{z}_G - \dot{z}_G) + mg \tag{8}$$

where ${}^{ref}z_G$ is the referential COG in z-axis, whose meaning varies depending on the cases as is mentioned later. K_{Pz} and K_{Dz} should be chosen properly in accordance with the contact state and the motion scheme as is shown in Fig.2. I, II and III in Fig.2 are described as follows.

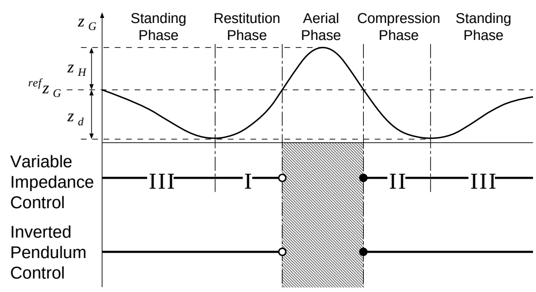


Figure 2: Contact phase, impedance and inverted pendulum control

I)Impedance for lift-off – restitution phase

It is required that COG is given an enough velocity and acceleration against the gravity in the case of lifting-off.

Then, K_{Pz} and K_{Dz} are designed as:

$$K_{Dz} = 0 \tag{9}$$

$$K_{Pz} = \frac{2mgz_H}{z_d^2} \tag{10}$$

where z_H is a planned maximum height from ${}^{ref}z_G$ in aerial phase and z_d is a planned maximum stooping depth from ${}^{ref}z_G$. In this case, it is expected that the robot will lift-off when z_G equals to ${}^{ref}z_G$.

II)Impedance for touchdown — compression phase

In order to reduce the shock given to the robot at the moment of touchdown, K_{Pz} and K_{Dz} are designed as:

$$K_{Dz} = 0 \tag{11}$$

$$K_{Pz} = \frac{m\dot{z}_{G-}^2}{z_d^2} \tag{12}$$

where ${}^{ref}z_G$ is the COG height at the moment of touchdown, $\dot{z}_{G-}$ is the falling speed immediately before touchdown, and z_d is a planned maximum stooping depth from ${}^{ref}z_G$ after touchdown.

III)Impedance in standing phase

The COG height z_G converges to ${}^{ref}z_G$ without overshoot when K_{Pz} and K_{Dz} satisfy the following condition.

$$K_{Pz} > 0, \quad K_{Dz} > 0, \quad K_{Dz}^2 - 4K_{Pz} > 0 \tag{13}$$

And to conserve a continuous contact to the ground, ${}^{ref}f_z$ must satisfy

$${}^{ref}f_z > 0 \tag{14}$$

Thus, there should be an appropriate minimum limit ${}^{ref}f_{z,min}(> 0)$.

3 Unified Motion Control in Contact/Aerial Phase

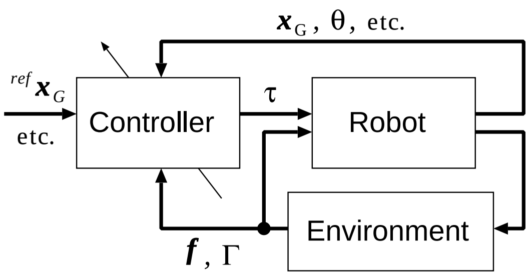


Figure 3: Humanoid control system connected with environment

Since humanoid robots are driven through interaction with the environment, the controlling system of them should include the environment within as is figured in Fig.3. Information is fedback not only from the inner parameters of them but from the environment. Based on this idea, how to design the controller is noted in this section.

3.1 Indirect Manipulation of ZMP and Vertical Reaction Force in Contact Phase

In the previous section, it has been shown that COG can be controlled and seamless transition between contact and aerial phases can be achieved through the manipulation of ZMP and vertical reaction force. It is impossible, however, to manipulate ZMP and vertical reaction force directly since the robot has an underactuated link in nature. This section describes how to calculate the equivalent inner force to manipulate them indirectly,

I) Strict Referential Velocity(SRV) of COG

When the acceleration ${}^{ref}\ddot{\boldsymbol{x}}_G$ in (15)(16)(17) is given to COG instantaneously, ZMP and vertical reaction force coincide with ${}^{ref}\boldsymbol{x}_Z$ and ${}^{ref}f_z$ respectively, in accordance with the equations (3)(4)(5).

$${}^{ref}\ddot{x}_G = {}^{ref}\omega_G^2(x_G - {}^{ref}x_Z) - \frac{n_y}{m(z_G - {}^{ref}z_Z)} \tag{15}$$

$${}^{ref}\ddot{y}_G = {}^{ref}\omega_G^2(y_G - {}^{ref}y_Z) + \frac{n_x}{m(z_G - {}^{ref}z_Z)} \tag{16}$$

$${}^{ref}\ddot{z}_G = \frac{{}^{ref}f_z}{m} - g \tag{17}$$

where ${}^{ref}\ddot{\boldsymbol{x}}_G = \left[{}^{ref}\ddot{x}_G \quad {}^{ref}\ddot{y}_G \quad {}^{ref}\ddot{z}_G \right]^T$ and ${}^{ref}\omega_G$ is defined as:

$${}^{ref}\omega_G^2 = \frac{{}^{ref}f_z}{m(z_G - {}^{ref}z_Z)} \tag{18}$$

and n_x and n_y are the current moment around x-axis and y-axis respectively. It requires, however, a large amount of computation to calculate the equivalent inner force to the above acceleration, so that it is difficult to realize a responsive motion which should be controlled in quite a short period.

Here, we integrate them and obtain the instantaneous strict referential velocity(SRV) of COG ${}^{ref}\dot{\boldsymbol{x}}_G$. It pragmatically reduces the amount of computation with the following procedure.

II) The Whole-body Motion Realizing SRV of COG

How to decompose the SRV of COG ${}^{ref}\dot{\boldsymbol{x}}_G$ to the equivalent whole-body motion ${}^{ref}\dot{\boldsymbol{\theta}}$ is described in this subsection, where $\boldsymbol{\theta}$ is a joint angle vector ($n\times 1$, n:number of the robot joints) and ${}^{ref}\boldsymbol{\theta}$ is the referential value of $\boldsymbol{\theta}$.

Since COG $\boldsymbol{x}_G$ is the function with an argument $\boldsymbol{\theta}$, there is a Jacobian $\boldsymbol{J}_G$ as:

$$\dot{\boldsymbol{x}}_G = \frac{\partial \boldsymbol{x}_G}{\partial \boldsymbol{\theta}} \dot{\boldsymbol{\theta}} \equiv \boldsymbol{J}_G \dot{\boldsymbol{\theta}} \tag{19}$$

We call this $\boldsymbol{J}_G$ *COG Jacobian* hereafter.

$\boldsymbol{J}_G$ is a quite complex non-linear function with multiple arguments. Tamiya et al. proposed the numerical method to calculate it [16], which needs a large amount of computation and is less accurate. We developed a fast and accurate calculation method of $\boldsymbol{J}_G$ with the numerical approach as follows.

Firstly, The relative COG velocity with respect to the total body coordinate(which moves with the base link of the robot together) ${}^0\dot{\boldsymbol{x}}_G$ can be expressed as:

$${}^0\dot{\boldsymbol{x}}_G = \frac{\sum_{i=0}^{n-1} m_i {}^0\dot{\boldsymbol{r}}_i}{\sum_{i=0}^{n-1} m_i} = \frac{\sum_{i=0}^{n-1} m_i {}^0\boldsymbol{J}_{Gi} \dot{\boldsymbol{\theta}}}{\sum_{i=0}^{n-1} m_i} \tag{20}$$

where m_i is the mass of link i, ${}^0\boldsymbol{r}_{G,i}$ is the position of the center of mass of link i with respect to the total body coordinate, and ${}^0\boldsymbol{J}_{G,i}$ $(3\times n)$ is defined by

$${}^0\boldsymbol{J}_{G,i} \equiv \frac{\partial {}^0\boldsymbol{r}_{G,i}}{\partial \boldsymbol{\theta}} \tag{21}$$

${}^0\boldsymbol{J}_G$ is calculated easily by the method proposed by Orin et al.[17]

Therefore, Jacobian ${}^0\boldsymbol{J}_G$ which relates $\dot{\boldsymbol{\theta}}$ to ${}^0\dot{\boldsymbol{x}}_G$ is

$${}^0\boldsymbol{J}_G = \frac{\sum_{i=0}^{n-1} m_i {}^0\boldsymbol{J}_{Gi}}{\sum_{i=0}^{n-1} m_i} \tag{22}$$

Secondly, suppose link F is fixed in the world coordinate (for example, when the right leg is the supporting leg, the right foot link is fixed), the COG velocity with respect to the world coordinate $\dot{\boldsymbol{x}}_G$ is

$$\begin{aligned} \dot{\boldsymbol{x}}_G &= \dot{\boldsymbol{x}}_0 + \boldsymbol{\omega}_0 \times \boldsymbol{R}_0 {}^0\boldsymbol{x}_G + \boldsymbol{R}_0 {}^0\dot{\boldsymbol{x}}_G \\ &= \boldsymbol{R}_0 \{ {}^0\dot{\boldsymbol{x}}_G - {}^0\dot{\boldsymbol{p}}_F + ({}^0\boldsymbol{x}_G - {}^0\boldsymbol{p}_F) \times {}^0\boldsymbol{\omega}_F \} \\ &= \boldsymbol{R}_0 \{ {}^0\boldsymbol{J}_G - {}^0\boldsymbol{J}_F + [({}^0\boldsymbol{x}_G - {}^0\boldsymbol{p}_F)^\times] {}^0\boldsymbol{J}_{\omega F} \} \dot{\boldsymbol{\theta}} \end{aligned} \tag{23}$$

where $\boldsymbol{x}_0$ is the position of the base link in the world coordinate, $\boldsymbol{\omega}_0$ is the rotation velocity of the base link with respect to the world coordinate, $\boldsymbol{R}_0$ is the attitude matrix of the base link with respect to the world coordinate, ${}^0\boldsymbol{p}_F$ is the position of the fixed link in the total body coordinate, ${}^0\boldsymbol{\omega}_F$ is the rotation velocity of the fixed link with respect to the total body coordinate, ${}^0\boldsymbol{J}_F$ is the Jacobian about linear velocity of the fixed link with respect to the total body coordinate, ${}^0\boldsymbol{J}_{\omega F}$ is the Jacobian about rotation velocity of the fixed link with respect to the total body coordinate, and the notation $[\boldsymbol{v}^\times]$ means outer-product matrix of a vector $\boldsymbol{v}$ (3×1).

Then, $\boldsymbol{J}_G$ can be calculated as:

$$\boldsymbol{J}_G = \boldsymbol{R}_0 \{ {}^0\boldsymbol{J}_G - {}^0\boldsymbol{J}_F + [({}^0\boldsymbol{x}_G - {}^0\boldsymbol{p}_F)^\times] {}^0\boldsymbol{J}_{\omega F} \} \tag{24}$$

Using $\boldsymbol{J}_G$, the constraint equation which ${}^{ref}\dot{\boldsymbol{\theta}}$ has to satisfy is expressed as:

$$\boldsymbol{J}_G {}^{ref}\dot{\boldsymbol{\theta}} = {}^{ref}\dot{\boldsymbol{x}}_G \tag{25}$$

III) Decomposition of Constraints to the Whole Joint Angle Motion

Some other constraints than the equation (25), which are about motions of extrimities, for example, would be imposed for task execution, being expressed as:

$$\boldsymbol{J}_C {}^{ref}\dot{\boldsymbol{\theta}} = \boldsymbol{c} \tag{26}$$

Solving (25) and (26) simultaneously with weighted pseudoinverse matrix, these constraints are decomposed to the referential velocities of the whole joint angles ${}^{ref}\dot{\boldsymbol{\theta}}$.

IV) Calculation of Joint Torques

Joint torque which make the joint angle follow ${}^{ref}\dot{\boldsymbol{\theta}}$ are calculated by each local feedback controller, such as simple PD control. As a result, the whole-body motion of the robot is generated.

3.2 Attitude and Posture Control in Aerial Phase

While the robot is in aerial phase, motion of COG is constrained by the momentum conservation law. Therefore, only the attitude and posture of it is controllable. In this case, however, motion of the whole joint angles are also constraint by the same type of equation with (26). Consequently, the referential joint angles and torques are decided by the same decomposition with that in contact phase.

Though both the attitude and posture of the robot at touchdown, as is mentioned by several, largely affects on the stability after landing, a systematic methodology for it is yet to be established.

Fig.4 shows the detail of the controller proposed, where $\boldsymbol{p}_i$ is the position and attitude of extrimities and is controllable in the world coordinate in contact phase, while only relative motions of them to the total body coordinate can be controlled in aerial phase, and Γ is an indicator of the contact state (contact/uncontact), which works as a switch for COG control. When Γ is off, namely, the robot doesn't contact with the environment, constraint about COG motion (25) is invalidated. Although constraints are validated or invalidated depending on the contact state, the decomposition method of the constraints to the whole body motion is invariant on it. That is to say, the controller proposed can treat motions in both contact phase and aerial phase seamlessly.

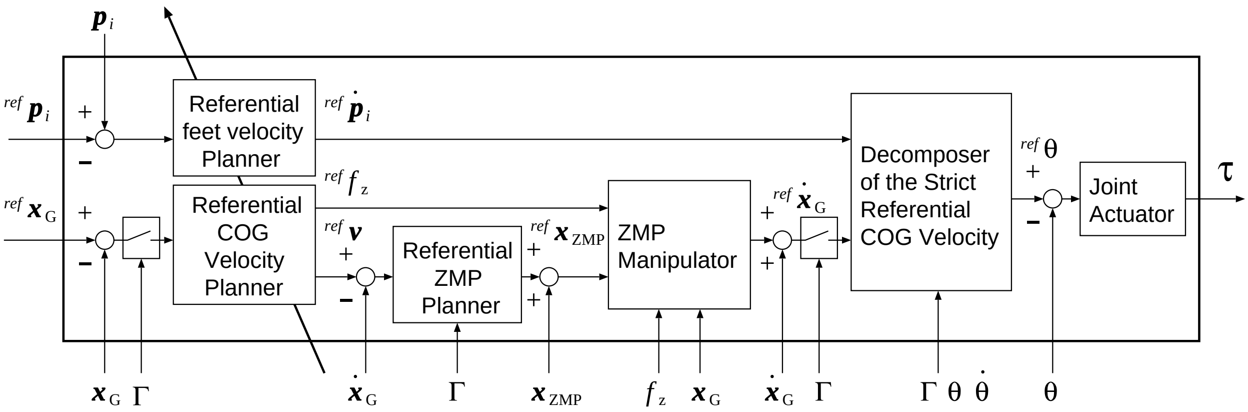
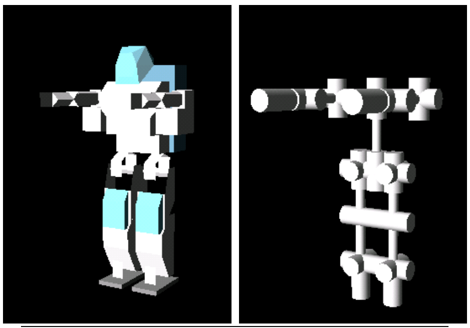


Figure 4: Block diagram of a humanoid controller

4 Simulation



DOF:	20 (8 for arm,12 for leg)
height:	480 [mm]
weight:	6.5 [kg]

Figure 5: Kinematic structure, size and mass of the robot

We realized a jumping motion in a computer simulation with the controller proposed, using a robot model of HOAP-1(Fujitsu Automation Ltd.)[18]. Kinematic structure, size and mass of the robot are shown in Fig.5.

The planned maximum stooping depth, detach-off height and the maximum height of jumping were 50[mm],220[mm],50[mm] respectively, and the referential height of COG after touchdown was 220[mm]. Impedance in each phase were decided in accordance with these factors. Fig.6 is a snapshot of the motion. And the loci of COG and ZMP are shown in Fig.7. We can make it sure that the impedance control method works properly and the stable jumping motion is achieved.

5 Conclusion

We proposed a new control method of humanoid robots based on the Variable Impedant Inverted Pendulum(VIIP) model, augmenting the method in [14].

The main faetures of proposed are

- it is invariant on contact phase and transition between contact/aerial phases is treated seamlessly. The transition are achieved with an adequate impedance.
- ZMP and the total external force acting to humanoid robots can be manipulated with a less amount of computation.

We verified the validity of the method with a computer simulation.

Acknowledgment

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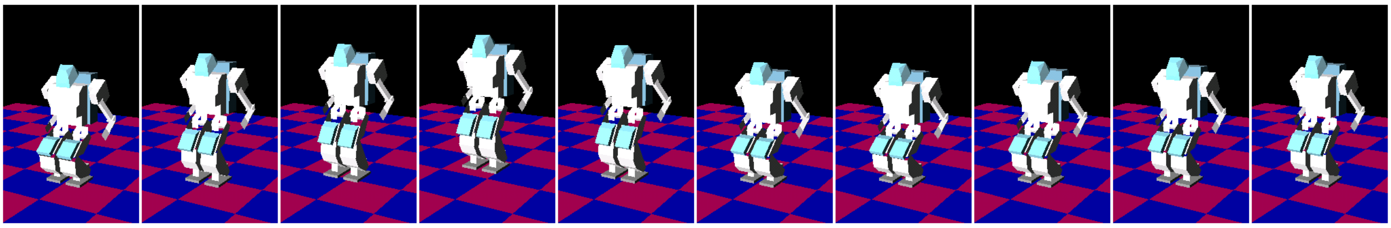

Figure 6: Snapshot of a jump motion simulation

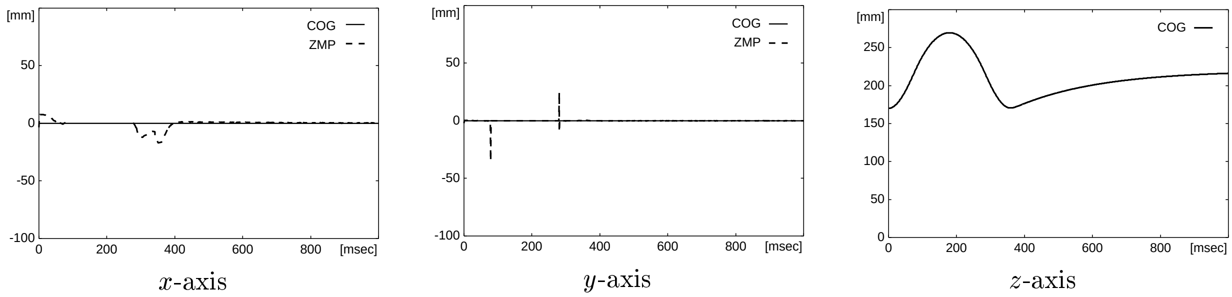

Figure 7: Loci of COG and ZMP in each axis

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