A Fast Online Gait Planning with Boundary Condition Relaxation for Humanoid Robots *

Tomomichi Sugihara and Yoshihiko Nakamura
Department of Mechano-informatics, Graduate school of University of Tokyo
7–3–1, Hongo, Bunkyo-ku, Tokyo, Japan
sugihara@ynl.t.u-tokyo.ac.jp

***Abstract*— A fast online gait planning method is proposed. Based on an approximate dynamical biped model whose mass is concentrated to COG, general solution of the equation of motion is analytically obtained. Dynamical constraint on the external reaction force due to the underactuation is resolved by boundary condition relaxation, namely, by admitting some error between the desired and actually reached state. It potentially creates responsive motion which requires strong instantaneous acceleration by accepting discontinuity of ZMP trajectory, which is designed as an exponential function. A semi-automatic continuous gait planning is also presented. It generates physically feasible referential trajectory of the whole-body only from the next desired foot placement. The validity of proposed is ensured through both simulations and experiments with a small anthropomorphic robot.**

***Index Terms*— humanoid robot, online biped gait planning, inverse kinematics**

I. Introduction

Efficacy of a pattern-based approach has been proved particularly for humanoid robots with a large number of degrees-of-greedom by many previous studies [1] [2] [3] [4] [5] [6]. It simplifies the strongly nonlinear control problem, explicitly breaking it into path-planning and stabilization. In the common manipulator control on this approach, a path from the initial state to the desired is planned at first by solving the *boundary value problem* in terms of geometry as Fig.1(A) shows. And then, a set of input torques to track the path is computed in terms of dynamics, if necessary. The primary difficulty of the motor control of legged robots, however, lies on the fact that path-planning and input computation – geometry and dynamics, in other words – are inseparable from each other under the severe constraints attributed to that abstracting forces are not generated at any contact points with the environment. It is since they lack of fixed base links to the inertial frame, and locomote through the conversion from the internal joint torques to the external reaction forces.

Vukobratović et al.[7] showed an iterative motion modification of the upper limb of a simple biped robot model so as to locate the center of the reaction force from the ground (Zero Moment Point[8], ZMP) at the tip of the supporting leg, for a given motion of the lower extremeties in offline. This idea was extended to more general styles by Yamaguchi et al.[2], Nagasaka et al.[3], Kitagawa et al.[4], Kajita et al.[6] and so forth. It basically replaces the path-planning problem of legged to the *initial value problem* for a given input as figured by Fig.1(B), and hence, it is not trivial that robots necessarily reach the desired goal. On the other hand, Kajita et al.[5], Nagasaka et al.[9] and Hasegawa et al.[10] analytically solved the piecewise equation of motion of an approximately mass-concentrated model, assuming the form of the input in each segment by zero-order-function, first-order-function and second-order-function, respectively, and calculated the coefficients from boundary condition. Park et al.[11] proposed GCIPM to take the effect of swing leg motion into account. They don't discuss the property of the input function, which essentially affects on whether it both satisfies the dynamical constraints and accelerates the robot as is needed. In addition, since Ref.[9] gives excess boundary condition, it is possibly an impossible problem when considering the beginning and end of the motion. Arakawa et al.[12], and Huang et al.[13] proposed trajectory planning methods which simultaneously realize satisfaction of dynamical constraints and achievement to the goal. They are not promising, using evolutional programming and hand tuning of interpolation parameters, respectively.

This paper proposes a fast motion planning of bipeds by a similar approach to Ref.[5][9][10] in the sense that it designs the trajectory with the analytical solution of the approximate equation of motion and boundary conditions. We focus on the facts that i) the robot locomotion does not necessarily require to reach the exact goal state, and that ii) discontinuity of the input, namely, the external force, at each end of segments is physically acceptable. Those properties enable completely online gait planning by relaxing boundary condition with dynamical constraints on the external force satisfied. The ZMP trajectory as the input is designed for an exponential function in order to achieve the skillful reaction force manipulation especially in agile and responsive types of motion for the exchange of supporting leg.

II. COG path planning with boundary condition relaxation

The strict equation of motion of a legged robot forms quite complicatedly with tens of degrees-of-freedom. locomotion of legged, however, is strongly dominated by the motion of the center of gravity(COG), and ignorance of the effect of moment around COG rarely leads to crises.

* This research was supported by FY2003 Abstracts for New Research Projects under Category "S" #15100002 of Grant-in-Aid for Scientific Research, Japan Society for the Promotion of Science.

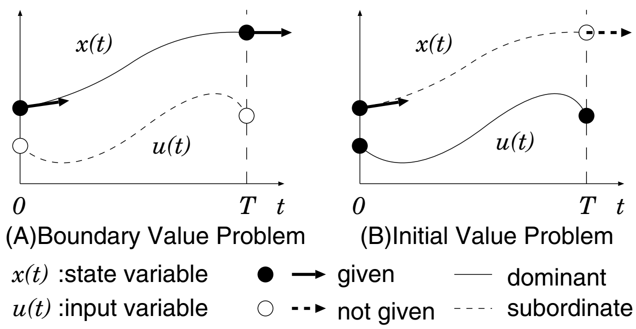


Fig. 1. Initial value problem versus boundary value problem

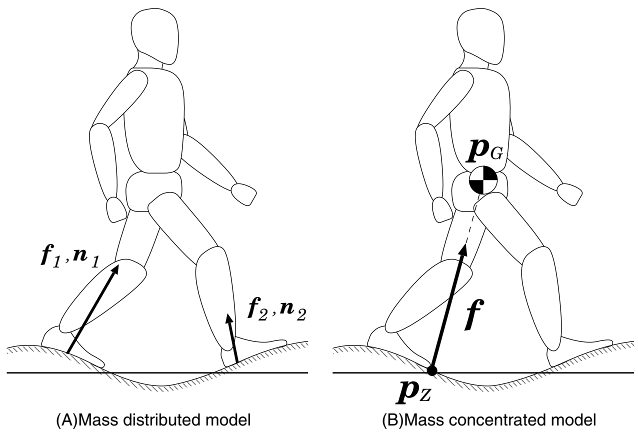


Fig. 2. Mass distributed model versus Mass concentrated model

The equation of motion of such approximate model with the total mass concentrated to COG as figure Fig.2(B) is expressed as follows.

$$m(\ddot{\boldsymbol{p}}_G + \boldsymbol{g}) = \boldsymbol{f} \tag{1}$$

$$(\boldsymbol{p}_G - \boldsymbol{p}_Z) \times \boldsymbol{f} = \boldsymbol{0} \tag{2}$$

where m is the total mass, $\boldsymbol{p}_G = [\ x_G\ \ y_G\ \ z_G\ ]^T$ is COG, $\boldsymbol{g} = [\ 0\ \ 0\ \ g\ ]^T$ is the acceleration of the gravity, $\boldsymbol{f} = [\ f_x\ \ f_y\ \ f_z\ ]^T$ is the total linear force exerted to the robot, and $\boldsymbol{p}_Z = [\ x_Z\ \ y_Z\ \ z_Z\ ]^T$ is ZMP. Note that z_Z is the level of the ground, and is known. Each component of (1)(2) is expanded as follows.

$$\ddot{x}_G = \omega_G^2 (x_G - x_Z) \tag{3}$$

$$\ddot{y}_G = \omega_G^2 (y_G - y_Z) \tag{4}$$

$$\ddot{z}_G = \frac{f_z}{m} - g \tag{5}$$

where

$$\omega_G \equiv \sqrt{\frac{\ddot{z}_G + g}{z_G - z_Z}} \tag{6}$$

In (3)(4)(5), x_Z,y_Z and f_z are thought to be a set of input to $\boldsymbol{p}_G$. Consequently, one can formulate the motion planning of this model as the computation of $\boldsymbol{p}_G(t)$ from $\boldsymbol{p}_G(0)$ to $\boldsymbol{p}_G(T)$ in $t = 0 \sim T$, and of the corresponding input $x_Z(t)$,$y_Z(t)$ and $f_z(t)$. In this paper, we don't deal with types of motion through aerial phase such as jumping and running. Then, the following constraints are posed on the input.

$$\boldsymbol{p}_Z \in \boldsymbol{S}(t) \tag{7}$$

$$f_z \geq 0 \tag{8}$$

where $\boldsymbol{S}(t)$ is a certain convex region which the contact points between the robot and the environment define on the horizontal plane $z = z_Z$ (typically, a convex hull of the points if all the contact points are on that plane), and transforms discontinuously. Both (7) and (8) come from that attracting forces are not generated at every contact points.

The vertical motion (5) is independent from the horizontal. The following is an example interpolation from $z_G(0)$ to $z_G(T)$.

$$z_G(t) = z_G(0) + (z_G(T) - z_G(0)) \left(\frac{t}{T} - \sin \frac{2\pi t}{T} \right) \tag{9}$$

If the distance from $z_G(0)$ to $z_G(T)$ is small, one can assume that the effect of the vertical movement on the horizontal motion in T is enough slight to be ignored, namely, ω_G in (3)(4), which in fact varies as time passes, is a constant in $0 \leq t \leq T$. Hereinafter, ω_G is substituted for $\omega_G(0)$ (or the meanvalue of $\omega_G(0)$ and $\omega_G(T)$), and simply denoted by ω_G. The homogeneous equations of (3) and (4) are linear, and therefore, the analytical solutions of them are obtained when $x_Z(t)$ and $y_Z(t)$ as certain functions of t are given. From the symmetricity of (3) and (4), we consider the motion only in x-axis, hereinafter.

Let us design the input, namely, the referential trajectory of ZMP, as the following exponential function.

$$x_Z(t) = x_Z(T) - (x_Z(T) - x_Z(0))\, e^{-\beta \omega_G t} \tag{10}$$

where

$$\beta > 0, \qquad \beta \neq 1 \tag{11}$$

The choice of this function possibly creates dynamic kicking motion (kicking at $x_Z(0)$ on the ground and pivoting around $x_Z(T)$) and discontinuous transition of support state rather easily. β determines a magnitude of the kick (the larger β one chooses, the faster ZMP travels). Then, the general solution of (3) forms as follows.

$$\begin{aligned} x_G(t) = {} & C_1 e^{\omega_G t} + C_2 e^{-\omega_G t} \\ & + \frac{x_Z(T) - x_Z(0)}{\beta^2 - 1} e^{-\beta \omega_G t} + x_Z(T) \end{aligned} \tag{12}$$

where C_1 and C_2 are unknown coefficients. Differentiating the above, we get:

$$\begin{aligned} \dot{x}_G(t) = {} & \omega_G C_1 e^{\omega_G t} - \omega_G C_2 e^{-\omega_G t} \\ & - \beta \omega_G \frac{x_Z(T) - x_Z(0)}{\beta^2 - 1} e^{-\beta \omega_G t} \end{aligned} \tag{13}$$

From these equations, the boundary conditions at $t = 0$ and $t = T$ are gathered into the following matrix form.

$$\boldsymbol{C} \begin{bmatrix} C_1 \\ C_2 \\ x_Z(0) \\ x_Z(T) \end{bmatrix} = \begin{bmatrix} x_G(0) \\ \dot{x}_G(0) \\ x_G(T) \\ \dot{x}_G(T) \end{bmatrix} \tag{14}$$

where

$$
\boldsymbol{C} \equiv \begin{bmatrix} 1 & 1 & -\frac{1}{\beta^2-1} & \frac{1}{\beta^2-1}+1 \\ \omega_G & -\omega_G & \frac{\beta\omega_G}{\beta^2-1} & -\frac{\beta\omega_G}{\beta^2-1} \\ e^{\omega_G T} & -e^{\omega_G T} & -\frac{e^{\beta\omega_G T}}{-\beta^2-1} & \frac{e^{-\beta\omega_G T}}{\beta^2-1}+1 \\ \omega_G e^{\omega_G T} & -\omega_G e^{\omega_G T} & \frac{\beta\omega_G e^{-\beta\omega_G T}}{\beta^2-1} & -\frac{\beta\omega_G e^{-\beta\omega_G T}}{\beta^2-1} \end{bmatrix} \tag{15}
$$

Assuming $\omega_G > 0$, $\boldsymbol{C}$ is a regular matrix, that is to say, the trajectory of both ZMP and COG is uniquely determined from the initial condition $x_G(0)$,$\dot{x}_G(0)$ and the boundary condition $x_G(T)$,$\dot{x}_G(T)$. In this case, (7) is not assured to be satisfied.

Let us reconsider the meanings of each parameter in (14). $x_G(0)$ and $\dot{x}_G(0)$ are the initial condition and must be satisfied strictly in terms of continuity of motion, while $x_G(T)$ and $\dot{x}_G(T)$ are the desired state after T and can accept a certain error. And, $x_Z(0)$ and $x_Z(T)$ are required to satisfy (7), while C_1 and C_2 are coefficients without any constraints. Then, we substitute (14) for the following quadratic programming.

$$
\frac{1}{2}(\boldsymbol{x} - {}^d\boldsymbol{x})^T \boldsymbol{Q}^{-1}(\boldsymbol{x} - {}^d\boldsymbol{x}) \longrightarrow \text{min.} \quad \text{subject to} \quad \boldsymbol{Dx} = \boldsymbol{s} \tag{*}
$$

where

$$
\boldsymbol{x} \equiv [x_Z(0)\ x_Z(T)\ x_G(T)\ \dot{x}_G(T)]^T \tag{16}
$$

$$
{}^d\boldsymbol{x} \equiv \left[{}^dx_Z(0)\ {}^dx_Z(T)\ {}^dx_G(T)\ {}^d\dot{x}_G(T)\right]^T \text{:the desired } \boldsymbol{x} \tag{17}
$$

$$
\boldsymbol{Q} \equiv diag\{q_i\} \qquad (i = 1 \sim 4) \tag{18}
$$

$$
\boldsymbol{D} \equiv \begin{bmatrix} 1 & 0 & -d_{33} & -d_{34} \\ 0 & 1 & -d_{43} & -d_{44} \end{bmatrix} \tag{19}
$$

$$
\boldsymbol{s} \equiv \begin{bmatrix} d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix} \begin{bmatrix} x_G(0) \\ \dot{x}_G(0) \end{bmatrix} \tag{20}
$$

And, $d_{ij}(i,j = 1 \sim 4)$ are components of the inverse matrix of $\boldsymbol{C}$.

$$
\boldsymbol{C}^{-1} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \tag{21}
$$

Solving the problem (*), we get:

$$
\boldsymbol{x} = {}^d\boldsymbol{x} - \boldsymbol{QD}^T(\boldsymbol{DQD}^T)^{-1}(\boldsymbol{D}{}^d\boldsymbol{x} - \boldsymbol{s}) \tag{22}
$$

C_1 and C_2 is determined by the following.

$$
\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \end{bmatrix} \begin{bmatrix} x_G(0) \\ \dot{x}_G(0) \\ x_G(T) \\ \dot{x}_G(T) \end{bmatrix} \tag{23}
$$

Locating ${}^dx_Z(0)$ and ${}^dx_Z(T)$ in the supporting region at $t = 0$ and $t = T$, respectively, with a sufficient margin, (7) is satisfied. Now, we get the referential trajectory of ZMP and COG which carries the robot close to the desired goal at $t = T$ with the dynamical constraints satisfied.

III. Semi-automatic gait planning from foot placement

This section describes an online semi-automatic gait planning including the starting and end motion only from the specified foot placement based on the previous simultaneous path-planning of ZMP and COG in $t = 0 \sim T$. The foot is treated as a tip point for the simplicity (and, it is easily extended to the robot with a sole, regarding a certain point within as the foot tip). Note that one should consider the constraint about supporting region in both x and y at the same time, though we discuss the motion only in x-axis.

Suppose the foot of the supporting leg (or, the supporting foot, hereinafter) and the foot of the leg to kick the ground (or, the kicking foot, hereinafter) are initially at $x_S(0)$ and $x_K(0)$, respectively. And, suppose the desired place of kicking foot at $t = T$ is $x_K(T)$. In the meantime, the supporting foot stays at the initial position, namely, $x_S(t) = x_S(0)$. Here, we set the desired goal of COG at the middle point of the position of supporting foot and the desired position of kicking foot as follows.

$$
{}^dx_G(T) = \frac{x_K(T) + x_S(T)}{2} \tag{24}
$$

And, we set the desired goal velocity of COG as follows.

$$
{}^d\dot{x}_G(T) = \begin{cases} \alpha_x\left({}^dx_G(T) - x_G(0)\right) & \text{(when the COG keeps moving)} \\ 0 & \text{(when the COG stops)} \end{cases} \tag{25}
$$

where α_x determintes the ratio of the boundary velocity of COG against the mean velocity of COG in T. Finaly, we set the desired boundary condition of ZMP as follows.

$$
{}^dx_Z(0) = x_K(0) \tag{26}
$$

$$
{}^dx_Z(T) = x_S(0) \tag{27}
$$

And then, the referential trajectory of ZMP and COG is computed from proposed in the previous section.

From the referential trajectory of ZMP, one can predict the time when the kicking foot can detach off the ground, namely, since $x_Z(t)$ is monotony, the robot can lift up the kicking foot after $x_Z(t)$ comes within the sole of the supporting foot. Suppose the time is $t = T_s$, and we plan the referential trajectory of the kicking foot in the horizontal direction, for example, as follows.

$$
x_K(t) = \begin{cases} x_K(0) & (0 \le t < T_s) \\ x_K(0) + \left({}^dx_K(T) - x_K(0)\right)\left(\frac{t}{T} - \sin\frac{2\pi t}{T}\right) & (T_s \le t < T) \end{cases} \tag{28}
$$

And the referential trajectory of it in the vertical direction $z_K(t)$, for example, as follows.

$$
z_K(t) = \begin{cases} z_K(0) & (0 \le t < T_s) \\ z_K(0) + \left({}^dz_K(T) - z_K(0)\right)\left(\frac{t}{T} - \sin\frac{2\pi t}{T}\right) + h_K\left(1 - \cos\frac{2\pi t}{T}\right) & (T_s \le t < T) \end{cases} \tag{29}
$$

where $h_K(>0)$ is the maximum foot-lifting height. When $t = T$, reset it as $t = 0$, and then, one can repetitively plan the referential trajectory of feet, and the continuous gait in online.

IV. Inverse kinematics including COG

This section shows inverse kinematics to compute the whole joint angle for planned position of COG and feet. Since this problem has a redundancy and hard to have analytical solution in general, Newton=Raphson's numerical method is applied.

Firstly, one can relate the whole joint angle movement explicitly to the COG movement by COG Jacobian $\boldsymbol{J}_G$[14] as follows.

$$\delta \boldsymbol{p}_G = \boldsymbol{J}_G \delta \boldsymbol{\theta} \tag{30}$$

where $\delta \boldsymbol{p}_G$ and $\delta \boldsymbol{\theta}$ are infinite small displacements of COG and joint angles, respectively. Note that this $\boldsymbol{J}_G$ implicitly includes the movement of base link. Thanks to it, one can regard COG apparently as a function whose arguments are only the acutuated joint $\boldsymbol{\theta}$. The similar idea is also applicable to the motion of constrained extremeties, including the kicking foot.

$$\delta \boldsymbol{p}_C = \boldsymbol{J}_C \delta \boldsymbol{\theta} \tag{31}$$

where $\boldsymbol{p}_C$ is a set of positions of constrained points in the extremeties, and $\delta \boldsymbol{p}_C$ is its infinite small displacement. $\boldsymbol{J}_C$ is the corresponding Jacobian and is calculated by a method in Ref.[15]. As the result, a combination of the motion of COG and extremeties are described by one equation as follows.

$$\delta \boldsymbol{p}_U = \boldsymbol{J}_U \delta \boldsymbol{\theta} \tag{32}$$

where

$$\boldsymbol{p}_U \equiv \begin{bmatrix} \boldsymbol{p}_G \\ \boldsymbol{p}_C \end{bmatrix}, \qquad \boldsymbol{J}_U \equiv \begin{bmatrix} \boldsymbol{J}_G \\ \boldsymbol{J}_C \end{bmatrix} \tag{33}$$

In Newton=Raphson's method, terms beyond linear of the Taylor series expansion are neglected in vicinity of the real root. Then,

$${}^d\boldsymbol{p}_U \simeq \boldsymbol{p}_U + \boldsymbol{J}_U({}^d\boldsymbol{\theta} - \boldsymbol{\theta}) \tag{34}$$

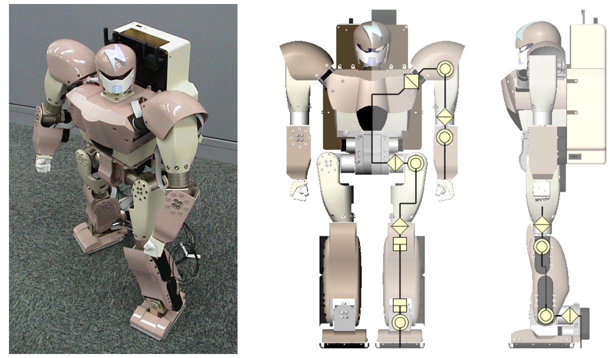
where ${}^d\boldsymbol{p}_U$ is the desired $\boldsymbol{p}_U$ and ${}^d\boldsymbol{\theta}$ is the corresponding whole joint angles to be the solution. From (34), $\boldsymbol{\theta}$ is updated according to the following rule, expecting to get $\boldsymbol{\theta}$ close to ${}^d\boldsymbol{\theta}$.

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{J}_U^{\star}({}^d\boldsymbol{p}_U - \boldsymbol{p}_U) \tag{35}$$

where $\boldsymbol{J}_U^{\star}$ is the Singularity-Robust Inverse matrix[16] to avoid numerical instability in iteration.

V. Example of gait generation and experiment

This section presents an example of the gait planning and the resultant motion through a simulation and an experiment. The external view, joint assignment and some specifications of the robot we used are shown in Fig.3.



Name:	UT-μ: mighty
DOF:	20
	8 for arms,12 for legs
height:	580 [mm]
weight:	6.5 [kg]

Fig. 3. External view, joint assignment and specifications of the robot

Tab.I is a series of the desired foot placements and the height of COG with time indices, where time is noted in second, and length in meter. Fig.4 shows the referential trajectory generated. One can note that a kicking motion to accelerate COG in the direction towards the supporting foot was achieved by exponential ZMP movement, while the resultant COG movement was smooth. Particularly, a rocking motion along y-axis was spontaneously created by its effect. At the end of the walking, COG didn't converge to the desired position only by exponential input trajectory of ZMP. In practice, a few times of repetitive replanning is required for a stopping motion, as the figure shows.

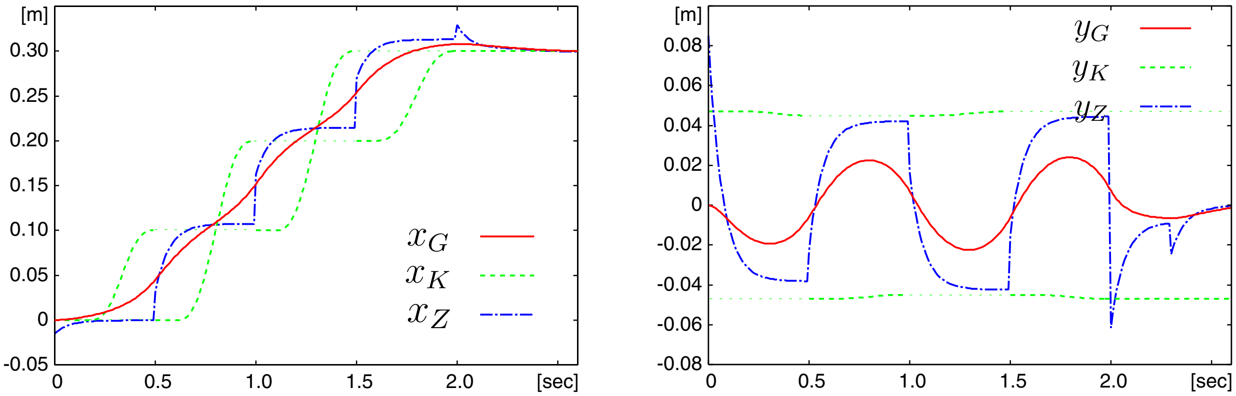
Fig.6 is a snapshot of a CG animation of the walking motion generated by applying inverse kinematics of the whole body. Fig.7 is a snapshot of a movie of the experiment using the real robot. In this experiment, each joint was controlled just as to follow the referential angle computed by proposed. In spite of that, a steady walking motion was achieved. Though the reaction forces from the ground were not measured, it succeeded to walk without force feedback, so that one can see the varidity of the trajectory planned.

The loci of the actual ZMP computed through inverse dynamics simulation and the planned are compared in Fig.5. The effect of moment around COG which is ignored in the planning phase appears as the error between them. Such disturbances might cause upsets of the real robot, so that they should be compensated. However, disturbances come in the real environment not only from the model error but also from lots of factors which are hard to be modeled such as uncertainty and dynamic variation of the circumstance, collision with unknown objects, and so forth. Hence, it is not realistic to aim at eliminating them in the planning stage. One should apply a reliable stabilization control [17].

VI. Conclusion

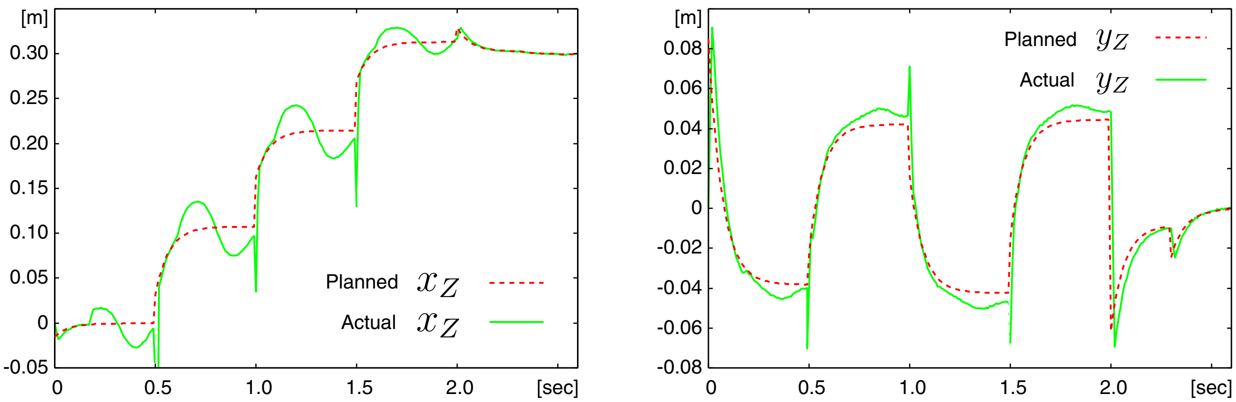
We developed a fast online gait planning method. The advantages of proposed are summarized to the following three items.

1) Dynamical inequality constraints about the external reaction force are satisfied by giving a relaxation to



(A) x-axis (B) y-axis

Fig. 4. Planned trajectory of COG, feet and ZMP



(A) x-axis (B) y-axis

Fig. 5. Planned ZMP and actual ZMP

TABLE I

A SERIES OF THE DESIRED FOOT PLACEMENTS AND THE HEIGHT OF COG AND TIME INDEX

T	switch†	${}^{d}x_K(T)$	${}^{d}y_K(T)$	${}^{d}z_K(T)$	${}^{d}z_G(T)$
0.7	L	0.10	0.045	0.0	0.28
0.7	R	0.20	-0.045	0.0	0.28
0.7	L	0.30	0.047	0.0	0.28
0.7	R	0.30	-0.047	0.0	0.29
0.3	S	0.30	0.000	0.0	0.29
0.2	S	0.30	0.000	0.0	0.29
0.1	S	0.30	0.000	0.0	0.29

† L ··· kicking with left foot,
R ··· kicking with right foot,
S ··· stop

the boundary condition. It stands on the property of each parameter, namely, the goal state does not necessarily have to coincide strictly with the desired in most cases of locomotion.

2) It allows discontinuity of ZMP in order to exclude any excess conditions. Moreover, it potentially creates responsive motion which requires strong instantaneous acceleration.
3) Exponential function is adopted for the design of the referential ZMP trajectory. It realizes the reaction force manipulation especially at the exchange of supporting foot in rather simple way. In addition, it is not required for motion designers to segment the motion explicitly into phases in terms of supporting state.

And, as an extension, a semi-automatic gait planning only from the next desired foot placement at each step was presented.

Although an approximate model of a biped whose mass is concentrated to COG substantially simplifies the dynamics of the biped robot so that general solution of the equation of motion is analytically obtained, the model error causes disturbances in turn. A combination with a reliable stabilization control is required in practice.

REFERENCES

[1] Kazuo Hirai, Masato Hirose, Yuji Haikawa, and Toru Takenaka. The Development of Honda Humanoid Robot. In *Proceeding of the 1998 IEEE International Conference on Robotics & Automation*, pages 1321–1326, 1998.

[2] Jin'ichi Yamaguchi, Eiji Soga, Sadatoshi Inoue, and Atsuo Takanishi. Development of a Bipedal Humanoid Robot – Control Method of Whole Body Cooperative Dynamic Biped Walking –. In *Proceedings of the 1999 IEEE International Conference on Robotics & Automation*, pages 368–374, 1999.

[3] Ken'ichiro Nagasaka. *The Whole Body Motion Generation of Humanoid Robot Using Dynamics Filter(in Japanese)*. PhD thesis, University of Tokyo, 2000.

[4] S. Kagami, T. Kitagawa, K. Nishiwaki, T. Sugihara, M. Inaba, and H. Inoue. A Fast Dynamically Equilibrated Walking Trajectory

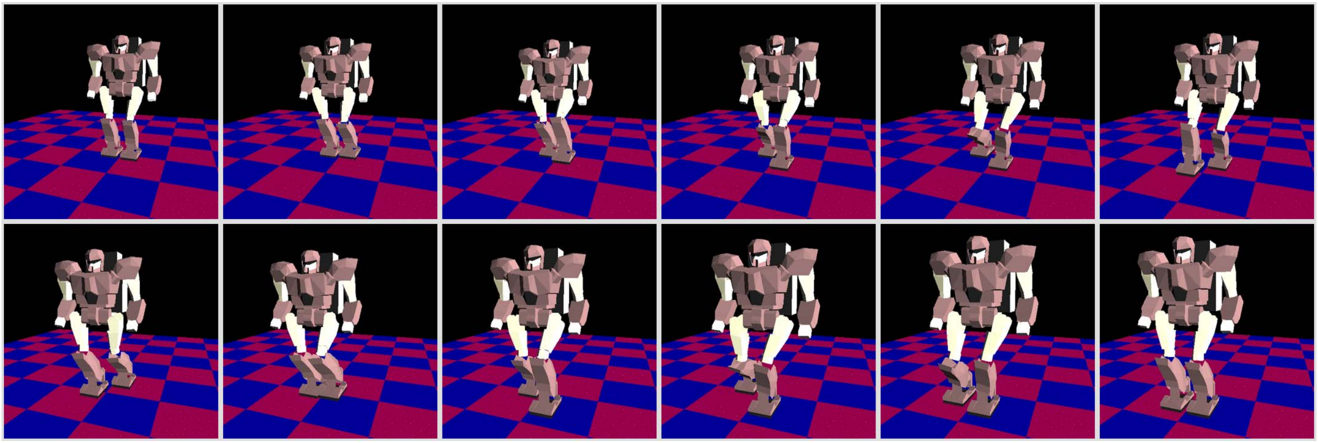


Fig. 6. Snapshots of the example walking motion generated

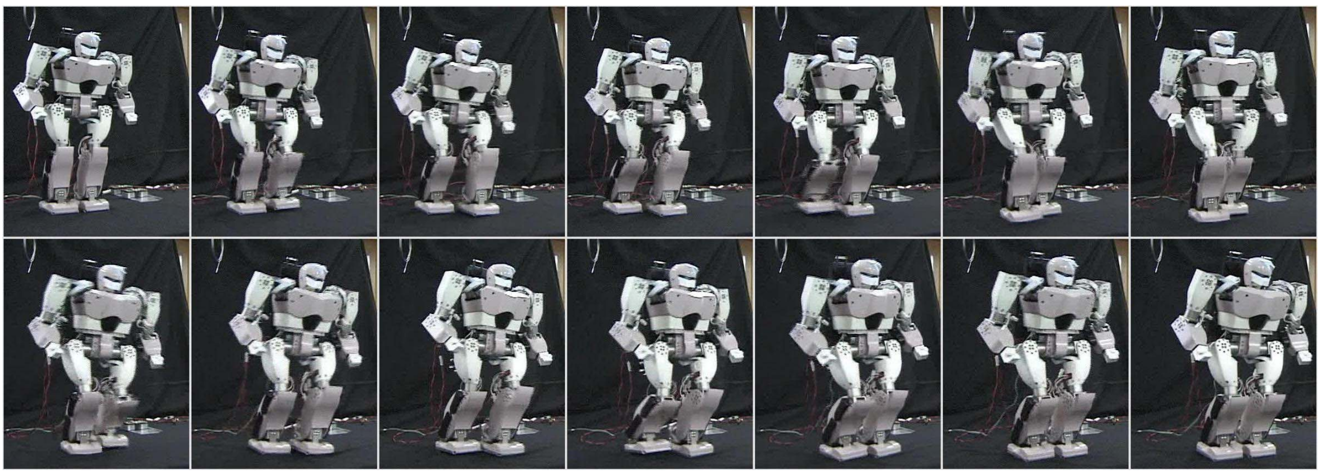


Fig. 7. Snapshots of the playbacked walking motion by the robot

Generation Method of Humanoid Robot. *Autonomous Robots*, 12(1):71–82, 2002.

[5] Shuuji Kajita, Osamu Matsumoto, and Muneharu Saigo. Real-time 3D walking pattern generation for a biped robot with telescopic legs. In *Proceedings of the 2001 IEEE International Conference on Robotics & Automation*, pages 2299–2036, 2001.

[6] Shuuji Kajita, Kensuke Harada, Fumio Kanehiro, Kenji Kaneko, Kiyoshi Fujiwara, Kazuhito Yokoi, and Hirohisa Hirukawa. A Dynamics Filter for ZMP Compensation by Preview Control(in Japanese). In *Proceedings of the 21st Annual Conference of the Robotics Society in Japan*, page 3A18, 2003.

[7] Miomir Vukobratović and Davor Juričić. Contribution to the Synthesis of Biped Gait. *IEEE Transactions on Bio-Medical Engineering*, BME-16(1):1–6, 1969.

[8] M. Vukobratović and J. Stepanenko. On the Stability of Anthropomorphic Systems. *Mathematical Biosciences*, 15(1):1–37, 1972.

[9] Ken'ichiro Nagasaka, Yoshihiro Kuroki, Shin'ya Suzuki, Yoshihiro Itoh, and Jin'ichi Yamaguchi. Integrated Motion Control for Walking, Jumping and Running on a Small Bipedal Entertainment Robot. In *Proceedings of the 2004 IEEE International Conference on Robotics and Automation*, pages 3189–3914, 2004.

[10] Tsutomu Hasegawa and Kan Yoneda. The Sway Compensation Trajectory for a Biped Robot. In *Proceedings of the 2003 IEEE International Conference on Robotics & Automation*, pages 925–931, 2003.

[11] Jong H. Park and Kyoung D.Kim. Biped Robot Walking Using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control. In *Proceedings of the 1998 IEEE International Conference on Robotics & Automation*, pages 3528–3533, 1998.

[12] Takemasa Arakawa and Toshio Fukuda. Natural Motion Generation of Biped Locomotion Robot using Hierarchical Trajectory Generation Method Consisting of GA, EP Layers. In *Proceedings of the 1997 IEEE International Conference on Robotics & Automation*, pages 211–216, 1997.

[13] Qiang Huang, Kazuhito Yokoi, Shuuji Kajita, Kenji Kaneko, Hirohiko Arai, Noriho Koyachi, and Kazuo Tanie. Planning Walking Patterns for a Biped Robot. *IEEE Transactions on Robotics and Automation*, 17(3):280–289, 2001.

[14] Tomomichi Sugihara, Yoshihiko Nakamura, and Hirochika Inoue. Realtime Humanoid Motion Generation through ZMP Manipulation based on Inverted Pendulum Control. In *Proceedings of the 2002 IEEE International Conference on Robotics & Automation*, pages 1404–1409, 2002.

[15] David E.Orin and William W.Schrader. Efficient Computation of the Jacobian for Robot Manipulators. *The International Journal of Robotics Research*, 3(4):66–75, 1984.

[16] Yoshihiko Nakamura. *ADVANCED ROBOTICS:Redundancy and Optimization*. Addison Wesley Publishing Company, 1991.

[17] Tomomichi Sugihara and Yoshihiko Nakamura. Whole-body Cooperative Balancing of Humanoid Robot using COG Jacobian. In *Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2002.