2009 IEEE International Conference on Robotics and Automation
Kobe International Conference Center
Kobe, Japan, May 12-17, 2009

Standing Stabilizability and Stepping Maneuver in Planar Bipedalism based on the Best COM-ZMP Regulator

Tomomichi Sugihara

Abstract— **The goal of this paper is to answer (i) how the stabilization performance of a biped controller can be evaluated on a certain invariant supporting region, (ii) how the standing stabilizer which performs the best on a given supporting region can be designed, (iii) how the system can be judged if it is stabilizable by the best standing stabilizer without deforming the current supporting region, and (iv) how the supporting region should be deformed if it is judged to be necessary. In order to answer these question, the stable standing region is defined. It gives a criterion to design the best standing stabilizer, to judge if the deformation of the supporting region is necessary to stabilize the system, and to maneuver the stepping motion in accordance with the standing stabilizability condition, which is also defined in the paper. It is found that the best standing stabilizer can be designed by a simple pole-assignment technique. This framework unifies the standing stability and the stepping stability of bipedalism, which have been separately considered in conventional studies. The discussion goes on an approximate planar COM-ZMP model, in which the total mass is concentrated at the center of mass, and the position of ZMP is regarded as the input. Though it is the simplest dynamical model of bipeds, it can conceal differences of body constitutions and represent the macroscopic dynamics. Therefore, this paper contributes to not only the biped robot controller design but also the biomechanical analyses.**

I. INTRODUCTION

The global stability of biped systems is a hard issue to discuss. It is because they are underactuated structure-varying systems[1] by nature due to the absence of mechanical connection to the environment. This property poses unilaterality and friction constraints on the reaction force, which the contact condition determines[2]. It is comparatively easier to discuss the stability as long as the supporting region is invariant. When considering the deformation of the region by stepping, the problem would become complicated.

Some criteria for walking stability have been proposed so far such as stationarity of body attitude, altitude and the gait[3][4][5], the existence of a limit cycle[6][7], and asymptotic convergence of discrete events to a periodic pattern[8][9][10][11]. Although these conventional studies have shed light on certain aspects of the bipedal motion, they are different from that in cases of standing motions. Pratt et al.[12] proposed *the capture point* to emboss the boundary of a region where a robot should step over to avoid falling down based on the orbital energy[13]. In their study, however, it is not clearly related to a standing control.

This work was supported in part by Grant-in-Aid for Young Scientists (B) #20760170, Japan Society for the Promotion of Science and by "The Kyushu University Research Superstar Program (SSP)", based on the budget of Kyushu University allocated under President's initiative.

T. Sugihara is with School of Information Science and Electrical Engineering, Kyushu University, 744 Moto'oka, Nishi-ku, Fukuoka, Japan zhidao@ieee.org

Further discussions with taking the area contact and the double-support phase into account are hard on a point-foot-contact model with an instantaneous exchange of the stance foot, which all of the above studies stand on. On the other hand, the past and present biped robots that walk via double-support phases with flat or multiple-point-contact soles have mainly targeted tracking controls to pre-planned trajectories [14][15][16][17], so that they have not deeply discussed the stability. We should also notice here that the center of mass (COM), the zero-moment point (ZMP)[18] and the reaction torque themselves at a certain instance can never give any information about the system stability as wrongly comprehended in many literatures[19][20][21][22][23]; they are not even necessary conditions for stability in fact.

The goal of this paper is to answer the following four questions, namely, (i) how the stabilization performance of a controller can be evaluated on a certain invariant supporting region, (ii) how the standing stabilizer which performs the best on a given supporting region can be designed, (iii) how the system can be judged if it is stabilizable by the best standing stabilizer without deforming the current supporting region, and (iv) how the supporting region should be deformed if it is judged to be necessary. In order to answer the question (i), *the stable standing region* is defined. The answer to (ii) is to maximize the stable standing region, which will be shown to be a simple pole-assignment problem. (iii) is achieved by utilizing the maximized stable standing region. The question (iv) is solved by introducing *the standing stabilizability condition*. It defines the future landing location of the stepping foot. Thus, this framework unifies the standing stability and the stepping stability of biped systems, which have been separately considered.

The above discussion goes on an approximate planar dynamical model whose total mass is concentrated at COM. The position of ZMP is regarded as the input. This idea was proposed by Mitobe et al.[24] and also by the authors[25]. It is similar to Kajita et al.'s model[13] but dissimilar at a point where COM can be controlled by continuous ZMP manipulation. Though it is the simplest model, it can conceal differences of body constitutions and can represent the macroscopic dynamics. Therefore, the conclusion drawn in this paper is applicable to not only the biped robot controller design but also the biomechanical analyses.

II. DESIGN OF THE BEST COM-ZMP REGULATOR

A. The COM-ZMP regulator

The strict equation of motion of fullbody biped dynamics is complicated with tens of degrees-of-freedom[2]. Here, we assume that an effect of inertial torque about COM is small

978-1-4244-2789-5/09/$25.00 ©2009 IEEE

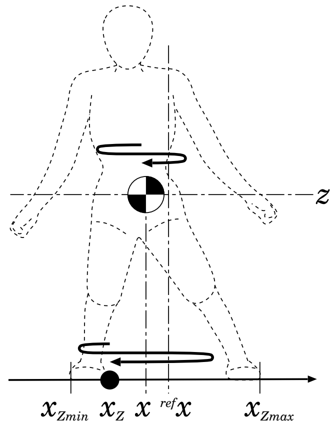

Fig. 1. An approximate mass-concentrated biped model in lateral plane. ZMP x_Z moves within the supporting region $x_{Zmin} \le x_Z \le x_{Zmax}$. The height of COM is assumed to be constant.

enough to be neglected, so that the macroscopic behavior of the system is approximately represented by a mass-concentrated COM-ZMP model. The equation of motion in lateral direction of a biped model is expressed as

$$\ddot{x} = \omega^2 (x - x_Z) \tag{1}$$

where x is the COM position, x_Z is the ZMP position and ω is defined as

$$\omega \equiv \sqrt{\frac{\ddot{z} + g}{z - z_Z}} \tag{2}$$

where z is the vertical COM height, z_Z is the ground level and g is the acceleration of gravity. Note that the inside of the square root is always non-negative in our cases because we don't consider suspended configurations. Hereafter, we assume that z is constant and $z_Z = 0$ as Fig.1, namely, $\omega = \sqrt{g/z}$, for simplicity. Then we get the following state-space representation of a linear system where ZMP position is regarded as the input:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega^2 \end{bmatrix} x_Z. \tag{3}$$

Even in this simplest system, x_Z is constrained to be within the supporting region as

$$x_{Zmin} \le x_Z \le x_{Zmax} \tag{4}$$

where x_{Zmin} and x_{Zmax} are the minimum and the maximum boundary of the supporting region, respectively.

Let us redefine the COM and ZMP coordinates with respect to the referential COM position ${}^{ref}x$ as

$$\chi \equiv x - {}^{ref}x \tag{5}$$

$$\chi_Z \equiv x_Z - {}^{ref}x. \tag{6}$$

We design the simulated ZMP[26] as

$$\tilde{\chi}_Z = -k_1 \chi - k_2 \dot{\chi}. \tag{7}$$

The actual ZMP is defined by a saturation rule as

$$\chi_Z = \begin{cases} \chi_{Zmax} & (\mathrm{S1} : \tilde{\chi}_Z > \chi_{Zmax}) \\ \tilde{\chi}_Z & (\mathrm{S2} : \chi_{Zmin} \le \tilde{\chi}_Z \le \chi_{Zmax}) \\ \chi_{Zmin} & (\mathrm{S3} : \tilde{\chi}_Z < \chi_{Zmin}) \end{cases} \tag{8}$$

where

$$\chi_{Zmin} \equiv x_{Zmin} - {}^{ref}x \tag{9}$$

$$\chi_{Zmax} \equiv x_{Zmax} - {}^{ref}x \tag{10}$$

with the constraint (4) taken into account. Then, we get a piecewise-affine autonomous system represented as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{cases} \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix}\begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega^2 \chi_{Zmax} \end{bmatrix} & (\mathrm{S1}) \\ \begin{bmatrix} 0 & 1 \\ \omega^2 (k_1 + 1) & \omega^2 k_2 \end{bmatrix}\begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} & (\mathrm{S2}) \\ \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix}\begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega^2 \chi_{Zmin} \end{bmatrix} & (\mathrm{S3}) \end{cases} . \tag{11}$$

Note that χ_{Zmin} and χ_{Zmax} are constants. Suppose the desired poles in S2 are given as ωq_1 and ωq_2. The feedback gains k_1 and k_2 are defined from them as

$$k_1 = -q_1 q_2 - 1, \quad k_2 = \frac{q_1 + q_2}{\omega}. \tag{12}$$

If $\Re q_1 < 0$ and $\Re q_2 < 0$, the COM state is regulated to the reference in the vicinity of $(\chi, \dot{\chi}) \simeq (0, 0)$. In this sense, let us call this controller the COM-ZMP regulator.

B. Definition of stable standing region

The piecewise-affine system (11) derived in the previous subsection can be visualized by solution curves in the phase space. Fig.2 shows a phase portrait for $\omega = \sqrt{g/0.27}$, ${}^{ref}x = 0$, $x_{Zmin} = -0.07$, $x_{Zmax} = 0.07$, and $(q_1, q_2) = (-0.2, -0.6)$. Fig.3 shows another portrait for the same ω, x_{Zmin}, x_{Zmax} and the set of poles $(q_1, q_2) = (-0.2, -2.0)$. In these figures, l_1, l_2, a and b are defined as

$$l_1 : \quad \chi + \frac{\dot{\chi}}{\omega} = \chi_{Zmin} \tag{13}$$

$$l_2 : \quad \chi + \frac{\dot{\chi}}{\omega} = \chi_{Zmax} \tag{14}$$

$$a : \quad -k_1 \chi - k_2 \dot{\chi} = \chi_{Zmin} \tag{15}$$

$$b : \quad -k_1 \chi - k_2 \dot{\chi} = \chi_{Zmax}. \tag{16}$$

The point A is the intersection of l_1 and a, and the point B is that of l_2 and b. When COM state $(\chi, \dot{\chi})$ is in the region between a and b, $\chi_Z \ne \mathrm{const.}$ so that the system is locally controllable. Thus, we call it *the controllable region*[6]. When $(\chi, \dot{\chi})$ is in the dotted area (blue area for readers with color), which is surrounded by l_1, l_2 and the phase trajectories passing A and B, COM stably converges to the reference on the current standing condition without deforming the supporting region. Hence, we define it as *the stable standing region*. Note that $(\chi, \dot{\chi}) = (\chi_{Zmin}, 0)$ and $(\chi, \dot{\chi}) = (\chi_{Zmax}, 0)$ are the singular points in the sense that, once COM is on l_1 or l_2, it converges to $(\chi_{Zmin}, 0)$ or $(\chi_{Zmax}, 0)$, respectively, and does not move toward the referential point. Hence the boundary is excluded from the stable standing region. Fig.4 shows a case where the reference is outside the supporting region. In this figure, the stable standing region doesn't exist. It is the necessary condition for the existence of the stable standing region that the reference is inside of the supporting region.

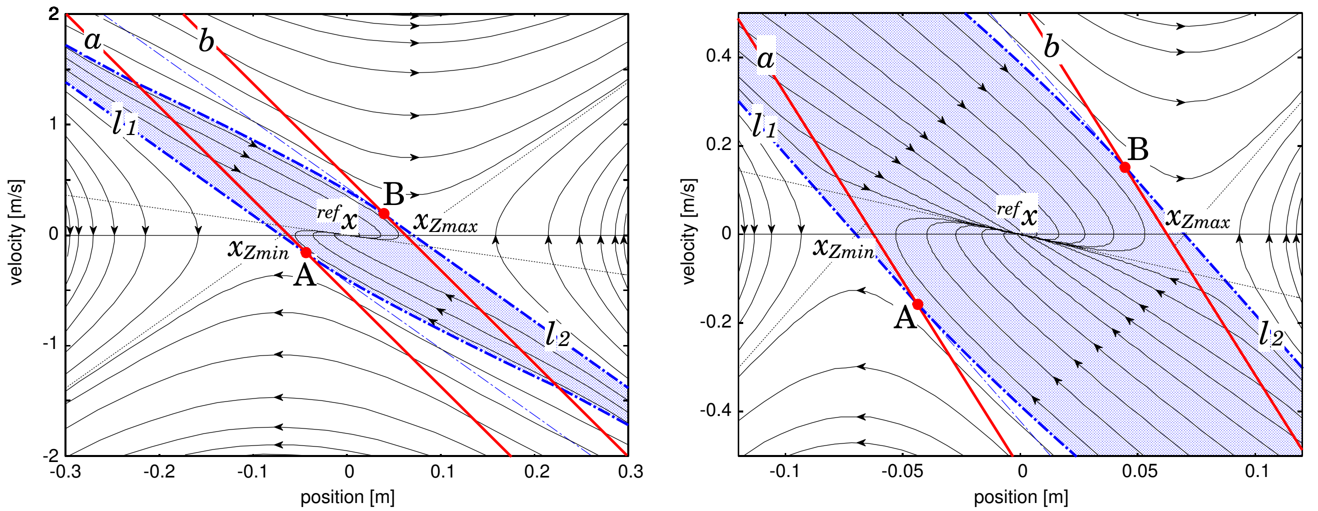

Fig. 2. Curves of the piecewise-affine autonomous system for $\omega = \sqrt{g/0.27}$, ${}^{ref}x = 0$, $x_{Zmin} = -0.07$ and $x_{Zmax} = 0.07$. Poles are assigned at -0.2ω and -0.6ω. The (blue) dotted area is the stable standing region. The right side is a zoomed-up area around the reference.

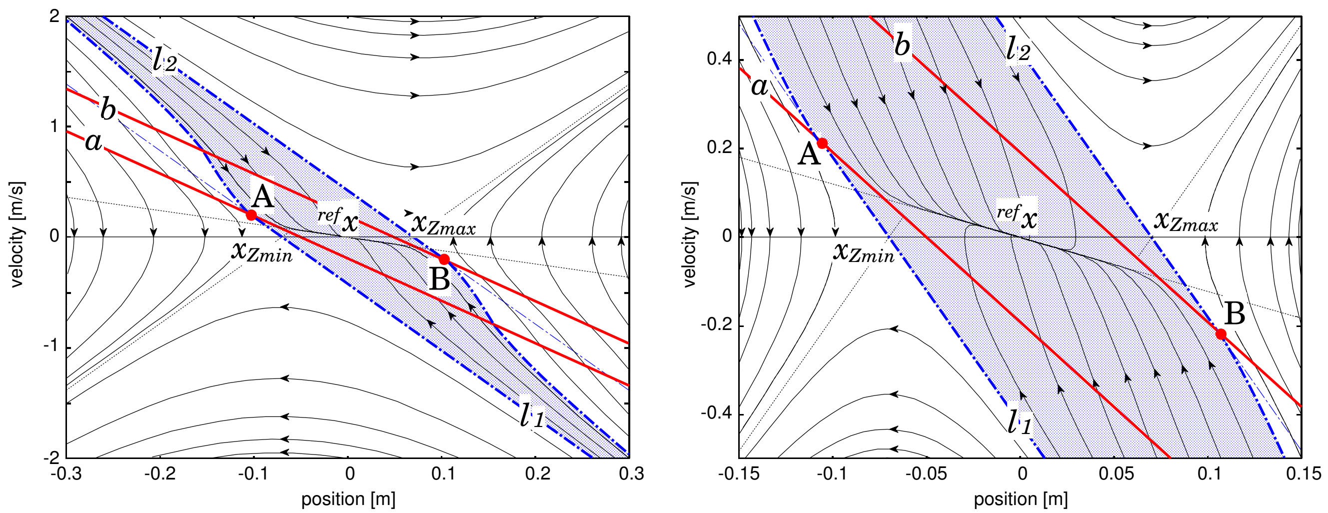

Fig. 3. Another curves of the piecewise-affine autonomous system for $\omega = \sqrt{g/0.27}$, ${}^{ref}x = 0$, $x_{Zmin} = -0.07$ and $x_{Zmax} = 0.07$. Poles are assigned at -0.2ω and -2.0ω. The (blue) dotted area is the stable standing region. The right side is a zoomed-up area around the reference.

C. The best COM-ZMP regulator and its condition

Performance of the COM-ZMP regulator can be evaluated based on the stable standing region. Obviously, the stable standing region is a subset of a region between l_1 and l_2, which we call *the confining region*, hereafter. The size of the stable standing region depends on the locations of A and B; the further from the singular points A and B are, the wider the stable standing region is. When a and b are parallel to l_1 and l_2, A and B vanish. The intersection of a and $\dot{\chi} = 0$ is $\left(\frac{\chi_{Zmin}}{q_1 q_2 + 1}, 0\right)$, and that of b and $\dot{\chi} = 0$ is $\left(\frac{\chi_{Zmax}}{q_1 q_2 + 1}, 0\right)$. When the reference is inside of the supporting region i.e. $\chi_{Zmin} < 0 < \chi_{Zmax}$, the following inequality is satisfied:

$$\chi_{Zmin} < \frac{\chi_{Zmin}}{q_1 q_2 + 1} < 0 < \frac{\chi_{Zmax}}{q_1 q_2 + 1} < \chi_{Zmax}. \tag{17}$$

It means that, if a and b are parallel to l_1 and l_2, a and b are included in the confining region and the stable standing region is maximized. It is the best standing stabilizer in the sense that it stabilizes any COM state which *could be* stabilized. Hence, we define *the best COM-ZMP regulator* as follows.

Definition 1. When the stable standing region with respect to a regulator coincides with the confining region, we call it *the best COM-ZMP regulator*.

In the case of the proposed piecewise-affine regulator represented by Eq.(7), the condition to design the best COM-ZMP regulator is rather easily obtained from Eq.(12) and Eq.(13)∼(16) as

$$\omega k_2 - k_1 = 0 \quad \Leftrightarrow \quad (q_1 + 1)(q_2 + 1) = 0. \tag{18}$$

Namely, if either q_1 or q_2 is equal to -1, the standing stabilization performance of the COM-ZMP regulator is the best. This simpler discussion than Morisawa et al. [27] is enabled by allowing non-differentiable ZMP manipulation. We assume $(q_1, q_2) = (-1, q)$, $(q < 0)$, hereafter. q can be an arbitrary negative real value, which should be designed with the actuators' performances, ex. the maximum speed,

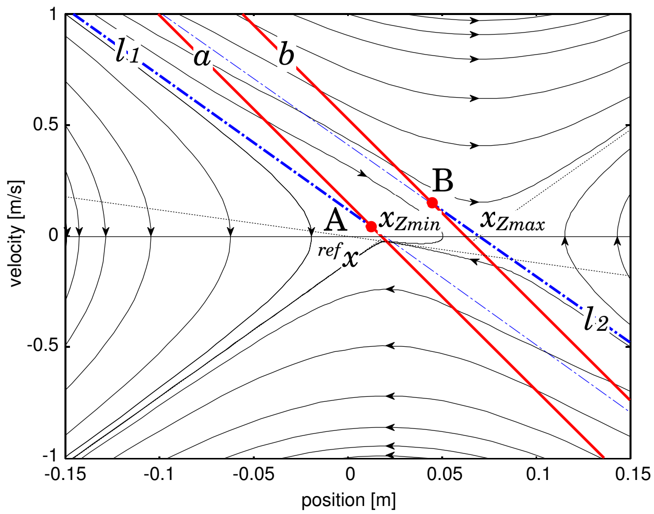

Fig. 4. Curves of the piecewise-affine autonomous system for $\omega = \sqrt{g/0.27}$, ${}^{ref}x = 0$, $x_{Zmin} = 0.02$ and $x_{Zmax} = 0.07$. Poles are assigned at -0.2ω and -0.6ω. The stable standing region does not exist, since the reference is outside of the supporting region.

taken into account. However, it is almost senseless if it is set for a less value than -1 since the system responsivity is decided from the dominant pole. In addition, (17) implies that less q makes the controllable region smaller.

The definition 1 is valid even for nonlinear regulators since the stable standing region in theory can never be larger than the confining region regardless of the controller properties.

Fig.5 is a phase portrait of the best COM-ZMP regulator with $q = -0.5$. It clearly shows that COM with an arbitrary initial condition inside of the confining region=the stable standing region converges to the reference.

III. Stabilizability condition and stepping

A. Stabilizability condition on the best COM-ZMP regulator

It is easy to judge if COM at a certain state can be stabilized by the best COM-ZMP regulator, since the stable standing region is only defined from l_1 and l_2 in the case. If the following condition is satisfied, COM is stabilizable:

$$\chi_{Zmin} < \chi + \frac{\dot{\chi}}{\omega} < \chi_{Zmax}. \tag{19}$$

This is equivalent with the capture point criterion[12]. Even in the stable standing region, however, COM displays a different behavior when it is inside or outside of the controllable region. Let us define the following two criteria.

Definition 2. If $\chi_{Zmin} < \chi + \frac{\dot{\chi}}{\omega} < \frac{\chi_{Zmin}}{1-q}$ or $\frac{\chi_{Zmax}}{1-q} < \chi + \frac{\dot{\chi}}{\omega} < \chi_{Zmax}$, COM is *weakly standing-stabilizable*.

Definition 3. If $\frac{\chi_{Zmin}}{1-q} < \chi + \frac{\dot{\chi}}{\omega} < \frac{\chi_{Zmax}}{1-q}$, COM is *strongly standing-stabilizable*.

B. Guarantee of future standing-stabilizability by stepping

When the condition (19) is not satisfied, it turns out that a stepping is required to deform the supporting region, and accordingly, the stable standing region.

Suppose $\chi + \frac{\dot{\chi}}{\omega} \leq \chi_{Zmin}$, which means that the minimum boundary of the supporting region χ_{Zmin} has to be modified. Let us denote it by χ^*_{Zmin}. If $\chi^*_{Zmin} < \chi + \frac{\dot{\chi}}{\omega}$, COM will be at least weakly standing-stabilizable with respect to the reformed supporting region i.e. COM will be within the stable standing region but not necessarily within the controllable region. If $\chi^*_{Zmin} < (1-q)\left(\chi + \frac{\dot{\chi}}{\omega}\right)$, COM will be strongly standing-stabilizable with respect to the new supporting region i.e. COM will be within the stable standing and controllable region. Not to mention, the reference has to be included in the reformed supporting region, namely, $\chi^*_{Zmin} < 0$ for both cases.

If $\chi + \frac{\dot{\chi}}{\omega} \geq \chi_{Zmax}$, the maximum boundary of the supporting region χ_{Zmax} has to be modified. Let us denote it by χ^*_{Zmax}. In that case, the weakly standing-stabilizable condition and the strongly standing-stabilizable condition after the landing are $\chi^*_{Zmax} > \chi + \frac{\dot{\chi}}{\omega}$ and $\chi^*_{Zmax} > (1-q)\left(\chi + \frac{\dot{\chi}}{\omega}\right)$, respectively.

By putting $(q_1, q_2) = (-1, q)$ into Eq.(12) and (7), we get

$$\tilde{\chi}_Z = (1-q)\left(\chi + \frac{\dot{\chi}}{\omega}\right) \tag{20}$$

as the simulated ZMP. It is the same with the boundary of the strongly standing-stabilizable condition. In other words, COM is strongly standing-stabilizable if the boundary of supporting region is located over the simulated ZMP of the best COM-ZMP regulator. It is preferable to satisfy the strongly standing-stabilizable condition as a stepping strategy. However, the above discussion implies that it is still safe to land where the weakly standing-stabilizable condition is satisfied. This knowledge can be utilized when the strongly standing-stabilizable condition of the future supporting region is too severe in terms of the robot responsivity or the limit of foot-reachable space.

IV. Simulations of stepping maneuver

We examined the stepping maneuver proposed in section III by some simulations with a COM-ZMP model conforming to a robot mighty[28] shown in **Fig. 6**. For all simulations, $\omega = \sqrt{g/0.27}$ and $(q_1, q_2) = (-1, -0.5)$.

Firstly, foot landing motions from a single-support condition were simulated. By setting the referential COM outside of the supporting region, a tumbling acceleration is initiated. If it is during the single-support phase, the swing foot will land to enlarge the supporting region based on the stabilizable condition. Fig.7 illustrates the situation.

The initial COM state was set for $(x, \dot{x}) = (0, 0)$, and the initial supporting region was $(x_{Zmin}, x_{Zmax}) = (0.01, 0.07)$ with the left foot in contact and the right foot in air. It was supposed to take 0.4[s] to land the right foot. Fig.8 shows the resultant loci of COM and ZMP. The left side is for a case where the right foot lands based on the weakly standing-stabilizable condition, while the right side is for a case where that is based on the strongly standing-stabilizable condition.

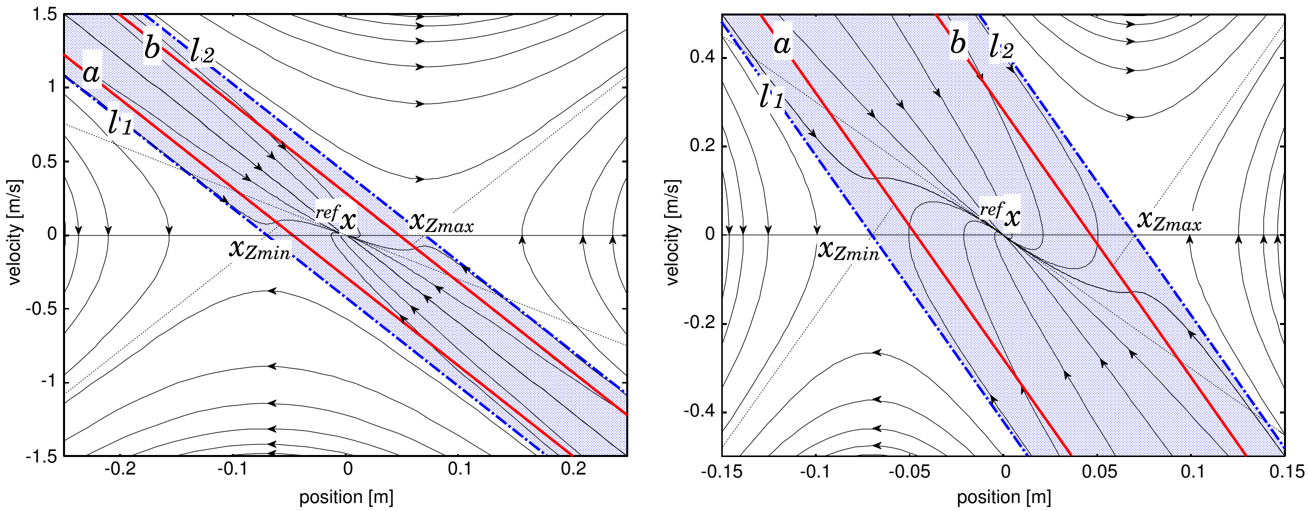

Fig. 5. Curves of the the piecewise-affine autonomous system for $\omega = \sqrt{g/0.27}$, ${}^{ref}x = 0$, $x_{Zmin} = -0.07$ and $x_{Zmax} = 0.07$ with the best COM-ZMP regulator. Poles are assigned at -1.0ω and -0.5ω. The stable standing region is maximized, coinciding with the confining region.

Height	: 0.58 [m]
Weight	: 6.5 [kg]
Number of joints	: 20 (8 for arms 12 for legs)
COM height in upright configuration	: 0.29 [m]

Fig. 6. External view and specifications of the supposed robot mighty.

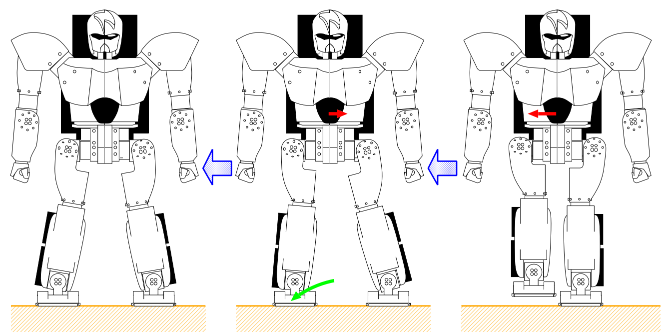

Fig. 7. When the referential COM is set outside of the supporting region, a tumbling acceleration is initiated. If it is during the single-support phase, the swing foot will land to enlarge the supporting region based on the stabilizable condition.

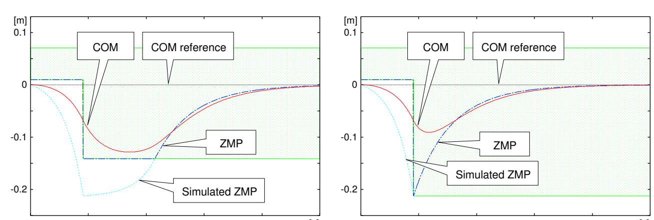

Fig. 8. Simulations of supporting region deformation by foot landing. Left: a foot landing based on weakly standing-stabilizable condition. Right: a foot landing based on strongly standing-stabilizable condition.

The latter result shows a better COM behavior with smaller deviation and faster convergence to the reference. It succeeded to gain sufficient accelerations to regulate COM with a larger supporting region after landing. However, the former result also shows a success to stabilize COM with about 67% area of the strongly standing-stabilizable supporting region. The displacement of the right foot for each case was about 0.07[m] and about 0.14[m], respectively.

Then, stepping-out motions from a double-support condition were simulated. When COM state is brought out of the stable standing region during the double-support phase, ZMP is once forced to jump into the pivot sole, and then the kicking foot steps out to enlarge the supporting region based on the stabilizable condition. Fig.9 illustrates the situation.

The initial COM state was set for $(x, \dot{x}) = (0, -0.45)$, and the initial supporting region was $(x_{Zmin}, x_{Zmax}) = (-0.07, 0.07)$ with both feet in contact. x_{Zmin} was set for 0.01 at 0.1[s] to jump ZMP jumped to 0.01, which emulates the situation where the right foot detached off the ground at that moment. After lifting off the right foot, the referential COM was updated to the midpoint of the both feet at every moment. It was supposed to take 0.3[s] to land the right foot based on the weakly standing-stabilizable condition. Fig.10 shows the resultant loci of COM and ZMP. The left side is for a success case where the final $x_{Zmin} = -0.27$, while the right side is for a failure case with 0.005[m] shortage of the final landing displacement $x_{Zmin} = -0.265$. It shows that the weakly standing-stabilizable condition provides the border of standing stabilization.

V. Conclusion

The stable standing region was proposed to evaluate the stabilization performance of a biped control on an invariant supporting region based on the COM-ZMP regulator. The

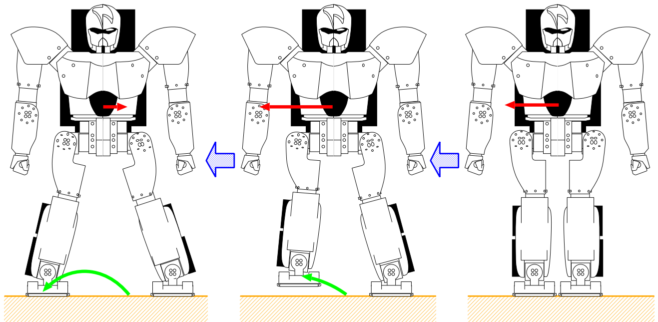

Fig. 9. When COM state is brought out of the stable standing region during the double-support phase, ZMP is forced to jump into the pivot sole, and then the kicking foot steps out to enlarge the supporting region based on the stabilizable condition.

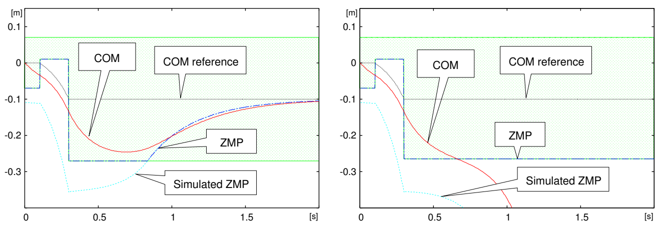

Fig. 10. Simulations of supporting region deformation by stepping-out. Left: a success case to stabilize COM based on weakly standing-stabilizable condition. Right: a failure case to stabilize COM because of mislanding.

condition to design the best COM-ZMP regulator, in which the stable standing region was maximized, was clarified. It does not only make it easy to judge if COM can be stabilized on the current supporting region, but also give a stepping maneuver to guarantee the future COM stabilizability. This framework can unify the standing stability and the stepping stability of bipedalism, which have been separately dealt with in conventional studies.

Although the discussion went on a mass-concentrated model with a constant COM height, the author thinks it can be enhanced to a mass-distributed model with variable COM height by using resolved COM rate control[25]. The next challenge is to enhance it to more dynamic three-dimensional motions including walks.

REFERENCES

[1] Y. Nakamura and K. Yamane, "Dynamics Computation of Structure-Varying Kinematic Chains and Its Application to Human Figures," *IEEE Transactions on Robotics and Automation*, vol. 16, no. 2, pp. 124–134, 2000.

[2] Y. Fujimoto, S. Obata, and A. Kawamura, "Robust Biped Walking with Active Interaction Control between Foot and Ground," in *Proceedings of the 1998 IEEE International Conference on Robotics & Automation*, 1998, pp. 2030–2035.

[3] M. Vukobratović, A. A. Frank, and D. Juričić, "On the Stability of Biped Locomotion," *IEEE Transactions on Bio-Medical Engineering*, vol. BME-17, no. 1, pp. 25–36, 1970.

[4] F. Gubina, H. Hemami, and R. B. McGhee, "On the Dynamic Stability of Biped Locomotion," *IEEE Transactions on Bio-Medical Engineering*, vol. BME-21, no. 2, pp. 102–108, 1974.

[5] J. Pratt and G. Pratt, "Intuitive Control of a Planar Bipedal Walking Robot," in *Proceedings of the 1998 IEEE International Conference on Robotics & Automation*, 1998, pp. 2014–2021.

[6] D. C. Witt, "A Feasibility Study on Automatically-Controlled Powered Lower-Limb Prostheses," University of Oxford," Report, 1970.

[7] R. Kato and M. Mori, "Control Method of Biped Locomotion Giving Asymptotic Stability of Trajectory," *Automatica*, vol. 20, no. 4, pp. 405–414, 1984.

[8] H. Miura and I. Shimoyama, "Dynamic Walk of a Biped," *The International Journal of Robotics Research*, vol. 3, no. 2, pp. 60–74, 1984.

[9] M. H. Raibert, H. B. B. Jr., and M. Chepponis, "Experiments in Balance with a 3D One-Legged Hopping Machine," *The International Journal of Robotics Research*, vol. 3, no. 2, pp. 75–92, 1984.

[10] T. McGeer, "Passive Dynamic Walking," *The International Journal of Robotics Research*, vol. 9, no. 2, pp. 62–82, 1990.

[11] J. W. Grizzle, G. Abba, and F. Plestan, "Asymptotically Stable Walking for Biped Robots: Analysis via Systems with Impulse Effects," *IEEE Transactions on Automatic Control*, vol. 46, no. 1, pp. 51–64, 2001.

[12] J. Pratt, J. Carff, S. Drakunov, and A. Goswami, "Capture Point: A Step toward Humanoid Push Recovery," in *Proceeding of the 2006 IEEE-RAS International Conference on Humanoid Robots*, 2006, pp. 200–207.

[13] S. Kajita, T. Yamaura, and A. Kobayashi, "Dynamic Walking Control of a Biped Robot Along a Potential Energy Conserving Orbit," *IEEE Transactions on Robotics and Automation*, vol. 8, no. 4, pp. 431–438, 1992.

[14] K. Hirai, M. Hirose, Y. Haikawa, and T. Takenaka, "The Development of Honda Humanoid Robot," in *Proceeding of the 1998 IEEE International Conference on Robotics & Automation*, 1998, pp. 1321–1326.

[15] A. Takanishi, Y. Egusa, M. Tochizawa, T. Takeya, and I. Kato, "Realization of Dynamic Walking Stabilized with Trunk Motion," in *ROMANSY 7*, 1988, pp. 68–79.

[16] K. Nagasaka, "The Whole Body Motion Generation of Humanoid Robot Using Dynamics Filter (in Japanese)," Ph.D. dissertation, University of Tokyo, 2000.

[17] T. Sugihara and Y. Nakamura, "Whole-body Cooperative Balancing of Humanoid Robot using COG Jacobian," in *Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2002, pp. 2575–2580.

[18] M. Vukobratović and J. Stepanenko, "On the Stability of Anthropomorphic Systems," *Mathematical Biosciences*, vol. 15, no. 1, pp. 1–37, 1972.

[19] R. B. McGhee and A. A. Frank, "On the Stability Properties of Quadruped Creeping Gaits," *Mathematical Biosciences*, vol. 3, pp. 331–351, 1968.

[20] K. Yoneda and S. Hirose, "Tumble Stability Criterion of Integrated Locomotion and Manipulation," in *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1996, pp. 870–876.

[21] Q. Huang, S. Kajita, N. Koyachi, K. Kaneko, K. Yokoi, H. Arai, K. Komoriya, and K. Tanie, "A High Stability, Smooth Walking Pattern for a Biped Robot," in *Proceedings of the 1999 IEEE International Conference on Robotics & Automation*, 1999, pp. 65–71.

[22] A. Goswami, "Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point," *The International Journal of Robotics Research*, vol. 18, no. 6, pp. 523–533, 1999.

[23] J. J. Kuffner, S. Kagami, M. Inaba, and H. Inoue, "Dynamically stable motion planning for humanoid robots," *Autonomous Robots*, vol. 12, no. 1, pp. 105–118, 2002.

[24] K. Mitobe, G. Capi, and Y. Nasu, "Control of Walking Robots based on Manipulation of the Zero Moment Point," *Robotica*, vol. 18, pp. 651–657, 2000.

[25] T. Sugihara, Y. Nakamura, and H. Inoue, "Realtime Humanoid Motion Generation through ZMP Manipulation based on Inverted Pendulum Control," in *Proceedings of the 2002 IEEE International Conference on Robotics & Automation*, 2002, pp. 1404–1409.

[26] T. Sugihara, "Simulated Regulator to Synthesize ZMP Manipulation and Foot Location for Autonomous Control of Biped Robots," in *Proceedings of the 2008 IEEE International Conference on Robotics & Automation*, 2008, pp. 1264–1269.

[27] M. Morisawa, S. Kajita, K. Kaneko, F. Kanehiro, S. Nakaoka, K. Harada, K. Fujiwara, and H. Hirukawa, "Motion Generation of Emergency Stop at Double Support Phase for Humanoid Robot by Pole Assignment (in Japanese)," *Journal of the Robotics Society of Japan*, vol. 26, no. 4, pp. 341–350, 2008.

[28] T. Sugihara, K. Yamamoto, and Y. Nakamura, "Hardware design of high performance miniature anthropomorphic robots," *Robotics and Autonomous System*, vol. 56, no. 1, pp. 82–94, 2007.