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Biped Control To Follow Arbitrary Referential Longitudinal Velocity based on Dynamics Morphing

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Abstract— **A novel biped control is proposed. Since it doesn't require referential motion trajectories defined by time, it can achieve longitudinal walking which is flexible enough to follow an arbitrary referential velocity given at random timing and to cope with unexpected external forces. The controller is developed based on the dynamics morphing, which is a framework to enable seamless transitions between various motions by continuously morphing the dynamical structure of the feedback system. Thus, it is compatible with the standing, the stepping-out for emergency, and so forth. Three key techniques are (i) morphing from the standing stabilizer with a stable equilibrium point to the velocity-follower which lacks any equilibrium points, (ii) a foot control maneuver which is automatically activated together with the morphing into the velocity following control, and (iii) automatic update of the referential position of COM for safety when going back to the standing stabilization. Although it is based on the same principle with the simulated regulator proposed by the author, the proposed controller is advantageous to it as it doesn't require an additional automaton to update the referential position of COM. The idea was examined through interactive computer simulations.**

I. INTRODUCTION

Biped robots are expected as mobilities particularly in complex environments which are not necessarily well-designed for robots. In realistic and the most probable situations, it is impossible to acquire detailed information about environments and events happening during the task execution in advance. Although a biped control scheme in which a robot tracks a referential motion trajectory defined by time is widely used as the most successful method[1][2][3][4][5][6], it obviously has the limitation to cope with uncertainties in environments and unexpected disturbances. Since biped robots are strongly nonlinear dynamical systems, which perform various motions such as standing, stepping and walking through discontinuous deformation of the supporting region by exchanging the stance foot, it is challenging to control them without detailed referential motion trajectories.

An idea to build an online trajectory planner in the total feedback loop[7][8][9][10][11] have attained notable results so far; several important operations including walking on unknown terrain[10], interactive navigations using joysticks[11], avoidance of falling down against certain magnitude of perturbations[9], etc. have been performed. However, they impose a priori constraints such as the order of stance foot alternation, the number of steps, the time to the next landing, and so forth, so that they are not sufficiently flexible in unexpected situations. On the other hand, a number of biped controls which don't require preplanned trajectories defined by time have been proposed [12] [13] [14] [15] [16] [17] [18] [19] [20]. They stand on unrealistic assumptions such as point-foot contacts, instantaneous exchanges of the stance foot, and so forth. More seriously, they are basically incompatible with the standing control.

The author[21] proposed a novel paradigm named *dynamics morphing* for designing a controller in which each motion is represented in a phase space with dynamical constraint taken into account and adjacent motions are connected by an interpolation of controllers. Based on it, a biped controller which unifies the standing stabilization[22], the stationary alternate stepping[21][23] and the spontaneous stepping-out for emergency[24] was developed. The goal of this paper is to enhance this controller to achieve longitudinal walking which can follow arbitrary referential velocities given at random timings and cope with unexpected external forces. In order to realize it, the following three key techniques are proposed: (i) morphing from the standing stabilizer with a stable equilibrium point to the velocity-follower which lacks any equilibrium points, (ii) a foot control maneuver which is automatically activated together with the morphing into the velocity following control, and (iii) automatic update of the referential position of COM for safety when going back to the standing stabilization.

The above (ii) is essentially based on the same principle with *the simulated regulator* proposed by the author[19]. However, the simulated regulator has the following three drawbacks, namely, (I) it requires an additional automaton which successively updates the referential position of COM for continued walking, (II) it designs both the longitudinal and the lateral controller in a symmetric way, and thus, it easily causes collisions of the both leg particularly in sideway, and (III) it requires too precise performances of the servo controller and the measurement of ZMP, since it adopts the rate of ZMP as the input to the system. The method proposed in this paper resolves all those problems.

II. COM–ZMP MODEL AND DYNAMICS MORPHING[23]

The dynamics morphing is a framework to design a controller which enables seamless transitions between various motions by continuously morphing the dynamical structure of the feedback system as the whole. The dynamical structures of each motion are designed based on the dynamical constraint and the motion context. Here, we are targeting a design of a biped robot controller upon this framework.

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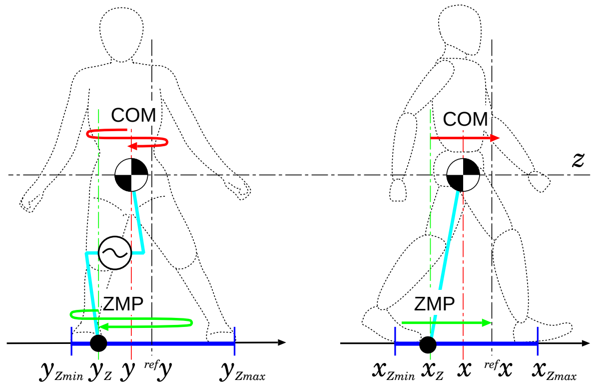


Fig. 1. COM–ZMP model

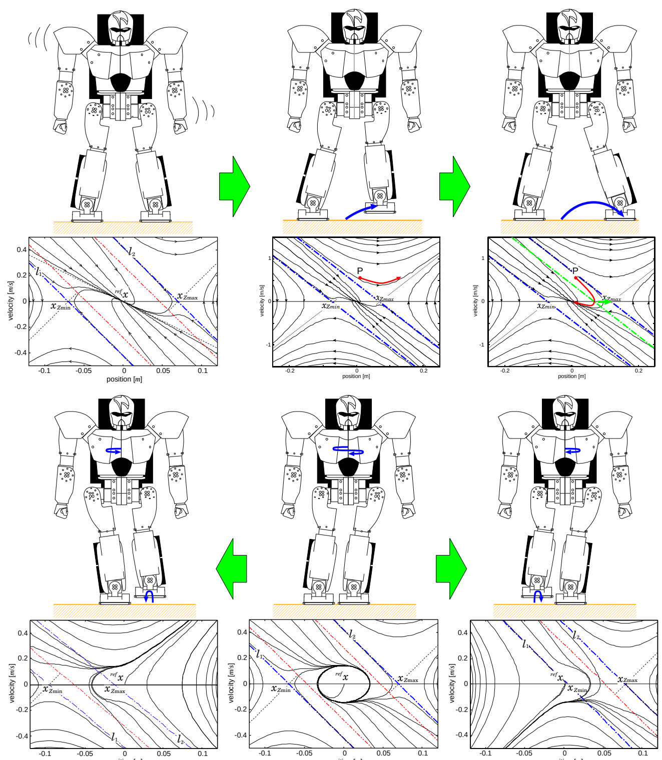


Fig. 2. Dynamics morphing unifying biped standing and stepping

Let us assign x, y and z axes along with the longitudinal, lateral and vertical directions, respectively, as depicted in Fig.1, and denote the location of the center of mass (COM) and the zero-moment point (ZMP)[25] by $\boldsymbol{p} = [x\ y\ z]^{\mathrm{T}}$ and $\boldsymbol{p}_Z = [x_Z\ y_Z\ z_Z]^{\mathrm{T}}$, respectively. Suppose the inertial torque about COM of the robot is sufficiently small to be ignored and the height of COM z is almost constant for simplicity, we get the following simplified equation of motion:

$$\ddot{x} = \omega^2(x - x_Z) \tag{1}$$

$$\ddot{y} = \omega^2(y - y_Z), \tag{2}$$

where $\omega \equiv \sqrt{g/z}$ and $g = 9.8[\mathrm{m/s^2}]$ is the acceleration due to the gravity. An important constraint that ZMP $\boldsymbol{p}_Z$ has to lie within the supporting region $\boldsymbol{S}$ is posed as

$$\boldsymbol{p}_Z \in \boldsymbol{S}. \tag{3}$$

The legged locomotion requires a combination of the ZMP manipulation within the supporting region $\boldsymbol{S}$ and the discontinuous deformation of $\boldsymbol{S}$. Biped robots should achieve it only by a pair of the left foot and the right foot.

Note that Eqs.(1) and (2) are symmetric with respect to x and y. Although one may think that the controllers for each direction could also be symmetric, it is inappropriate because of the following two reasons. One is that the midpoint of the both feet cannot be a stable equilibrium point when standing on one foot along y axis, while it can be along x axis. Another reason is that the both legs easily collide with each other in y direction, while it less happens in x direction.

For the motion along y axis i.e. the lateral direction, we have proposed the following biped control scheme so far. The desired ZMP as the control input is defined with respect to the referential position of COM ${}^{d}y$ as

$$\tilde{y}_Z = {}^{d}y + (q_y + 1)\left(y - {}^{d}y + f(\zeta)\frac{\dot{y}}{\omega}\right) \tag{4}$$

$$y_Z = \begin{cases} y_{Zmax} & (\mathrm{S1}: \tilde{y}_Z > y_{Zmax}) \\ \tilde{y}_Z & (\mathrm{S2}: y_{Zmin} \le \tilde{y}_Z \le y_{Zmax}) \\ y_{Zmin} & (\mathrm{S3}: \tilde{y}_Z < y_{Zmin}) \end{cases}, \tag{5}$$

where

$$f(\zeta) \equiv 1 - \rho \exp k \left\{1 - \frac{(q_y+1)^2\zeta^2}{r^2}\right\} \tag{6}$$

$$\zeta \equiv \sqrt{(y - {}^{d}y)^2 + \frac{\dot{y}^2}{\omega^2 q_y}}, \tag{7}$$

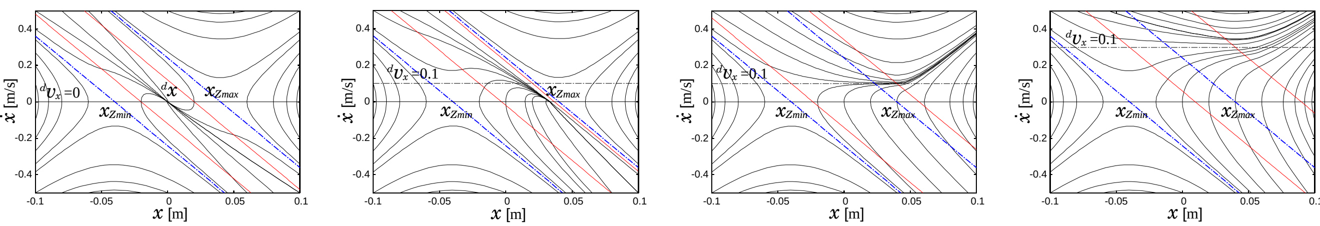
and $q_y(\ge 0)$, $k(>0)$, $r(>0)$ and $\rho(\ge 0)$ are constant parameters to be designed. Here, the supporting region $\boldsymbol{S}$ is simply represented by a segment $[y_{Zmin}, y_{Zmax}]$ along y axis. If the actual ZMP well tracks the desired ZMP, the motion of COM along y axis conforms the following piecewise autonomous system:

$$\ddot{y} = \begin{cases} \omega^2 y - \omega^2 y_{Zmax} & (\mathrm{S1}) \\ -\omega(q_y+1)f(\zeta)\dot{y} - \omega^2 q_y(y - {}^{d}y) & (\mathrm{S2}) \\ \omega^2 y - \omega^2 y_{Zmin} & (\mathrm{S3}) \end{cases}. \tag{8}$$

This controller works as the stabilizability-maximized COM–ZMP regulator [22] when $\rho = 0$, and it emerges the following stable limit cycle in state (S2) when $\rho > e^{-1}$,

$$(y - {}^{d}y)^2 + \frac{\dot{y}^2}{\omega^2 q_y} = \frac{(1 + \log \sqrt[k]{\rho})r^2}{(q_y+1)^2}. \tag{9}$$

In a particular case where $\rho = 1$, it represents a self-excited harmonic oscillator whose amplitude and period are $\dfrac{r}{q_y+1}$ and $\dfrac{2\pi}{\omega\sqrt{q_y}}$, respectively. In the stationary state, the corresponding ZMP synchronizes the oscillation with the amplitude r without phase lag. Based on this fact, an alternate stepping control in which the dynamical constraint about ZMP is automatically satisfied has also been presented[23]. Fig.2 illustrates the above scheme, in which the robot behavior is continuously morphed only by modifying ρ. See the referred paper for details. For the problem to track the desired ZMP via the whole body coordination, see Sugihara[26].



(a) $q_x = 1, {}^dv_x = 0$[m/s] (b) $q_x = 1, {}^dv_x = 0.1$[m/s] (c) $q_x = 0, {}^dv_x = 0.1$[m/s] (d) $q_x = 0, {}^dv_x = 0.3$[m/s]

Fig. 3. Dynamics morphing from a regulator to a velocity-follower by a modulation of the referential velocity and the pole-reassignment

III. Biped control to follow longitudinal referential velocity

A. Dynamics morphing from regulator to velocity-follower

Now, let us consider a control along x axis i.e. the longitudinal direction. For the standing stabilization, the same controlling scheme in which COM is regulated to the referential position is available for both x and y directions and it is not necessary to consider the discontinuous deformation of the supporting region. Suppose ${}^dx$ and ${}^dv_x$ are the referential position and velocity of COM along x axis, respectively, the regulator in x direction is designed as

$$\tilde{x}_Z = {}^dx + (q_x + 1)\left(x - {}^dx + \frac{\dot{x} - {}^dv_x}{\omega}\right) \tag{10}$$

$$x_Z = \begin{cases} x_{Zmax} & (\mathrm{T1}: \tilde{x}_Z > x_{Zmax}) \\ \tilde{x}_Z & (\mathrm{T2}: x_{Zmin} \le \tilde{x}_Z \le x_{Zmax}) \\ x_{Zmin} & (\mathrm{T3}: \tilde{x}_Z < x_{Zmin}) \end{cases}, \tag{11}$$

where the supporting region $\boldsymbol{S}$ is simply represented by a segment $[x_{Zmin}, x_{Zmax}]$ along x axis as well as that along y axis. It should be noted that this simplification will be resolved in the later examinations. The motion of COM along x axis conforms the following piecewise autonomous system:

$$\ddot{x} = \begin{cases} \omega^2 x - \omega^2 x_{Zmax} & (\mathrm{T1}) \\ -\omega(q_x{+}1)(\dot{x}{-}{}^dv_x){-}\omega^2 q_x(x{-}{}^dx) & (\mathrm{T2}) \\ \omega^2 x - \omega^2 x_{Zmin} & (\mathrm{T3}) \end{cases}. \tag{12}$$

In state (T2), the stationary point is $(x, \dot{x}) = \left({}^dx + \frac{q_x + 1}{\omega q_x}{}^dv_x, 0\right)$ for $q_x \neq 0$. It means that ${}^dv_x$ makes the stationary point shift from ${}^dx$, and doesn't make the COM velocity converge to ${}^dv_x$. Fig.3(a) is a phase portrait of the system for $q_x = 1$ and $({}^dx, {}^dv_x) = (0, 0)$, while Fig.3(b) is that for $q_x = 1$ and $({}^dx, {}^dv_x) = (0, 0.1)$. Readers may see that the stationary point in (b) is shifted from that in (a). On the other hand, the equation of motion in state (T2) for $q_x = 0$ turns to

$$\ddot{x} = -\omega(\dot{x} - {}^dv_x), \tag{13}$$

and the stationary point doesn't exist and $\dot{x}$ converges to ${}^dv_x$. Note that the time constant of the convergence only depends on ω. If ${}^dv_x > 0$, the state eventually goes into T1. If ${}^dv_x < 0$, it goes into T3. In whichever cases, $\dot{x}$ will diverge. Fig.3(c) is a phase portrait of the system for $q_x = 0$ and $({}^dx, {}^dv_x) = (0, 0.1)$. Fig.3(d) is that for $q_x = 0$ and $({}^dx, {}^dv_x) = (0, 0.3)$. They illustrate the above facts.

B. Consistent foot control with velocity-follower

The fact shown in the previous subsection leads to the following two discussions.

1) In the standing stabilization, it is senseless to give non-zero ${}^dv_x$; If non-zero ${}^dv_x$ is given, it should be interpreted that the velocity following control is required by the operator.
2) The velocity following control obviously requires the alternate stepping; if not for it, the robot would necessarily fall down. Hence, when non-zero ${}^dv_x$ is given, the lateral oscillation for the stepping[23] should be automatically activated.

Therefore, ${}^dv_x$, q_x and ρ, which have been independently tuned so far, should be linked. Namely, once a non-zero ${}^dv_x$ is given, (q_x, ρ) immediately before that is reserved as (q'_x, ρ'), and then, (q_x, ρ) is switched to $(0, 1)$ to begin stepping. When ${}^dv_x = 0$ is given again, (q_x, ρ) is reset for the reserved value (q'_x, ρ').

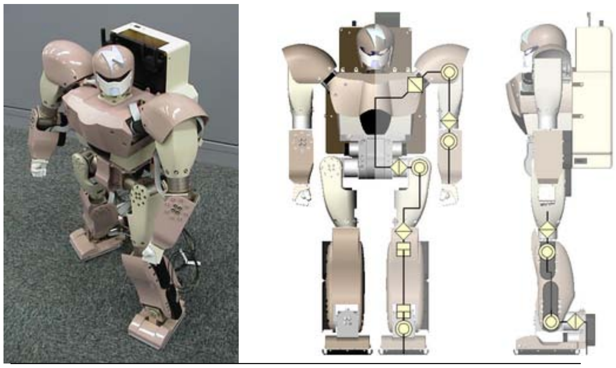
The feet along x axis are maneuvered in order to continue walking as follows. The desired landing position ${}^dx_S$ of the swinging foot is defined based on *the standing stabilizability condition*[22], and the system morphs again into the velocity-follower after the foot landing. Now, the strongly standing-stabilizable condition and the weakly standing-stabilizable condition introduced in the literature[22] are equivalent since $q_x = 0$, so that ${}^dx_S$ can be defined as

$${}^dx_S = x + \frac{\dot{x}}{\omega}. \tag{14}$$

Note that it never means that the robot can be stopped at any moment. The actual landing position of the swinging foot is determined rather in a discrete manner by the alternate stepping control synchronizing to the oscillatory behavior of ZMP in the lateral direction[23]. The motion is uncontrollable when the ZMP is stucked at the edge of the supporting region, whereas the dynamical constraint (3) is guaranteed to be satisfied. This maneuver is essentially the same with that based on *the simulated regulator*[19].

C. Automatic update of the referential position

In the velocity following control, the referential position of COM ${}^dx$ disappears from the system defined by Eq.(13) and the robot moves without relation to ${}^dx$. Thus, it is dangerous particularly at the moment when going back to the standing stabilization if ${}^dx$ remains even during walking, since it causes an inconsistency with the current location of COM



Name:	mighty
Height:	580 [mm]
Weight:	6.5 [kg]
Number of joints:	20 (8 for arms,12 for legs)

Fig. 4. External view of the robot mighty and its joint assignment

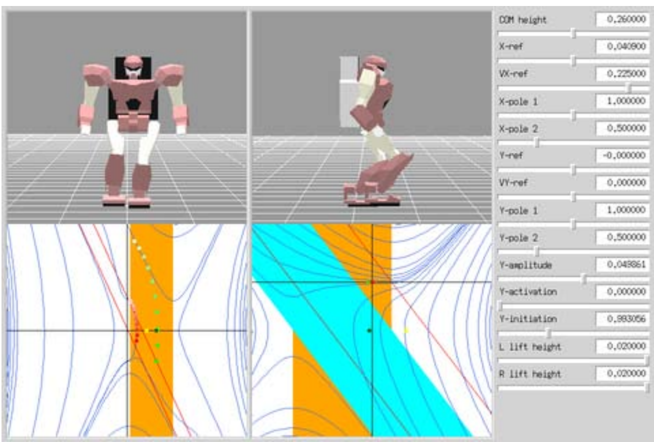


Fig. 5. Screen shot of the simulation program enabling interactive controls

and the supporting region. Then, it should be reset for a position upon the current supporting region.

According to the maneuver introduced in the previous subsection, the robot is taken back to the standing stabilization by giving ${}^{d}v_x = 0$. In order to make the robot stop after that, let us update the referential position of COM by the current position during ${}^{d}v_x \neq 0$, namely, ${}^{d}x = x$.

IV. Simulation

A simulation of a forward/backward walk was conducted based on the specification of a miniature anthropomorphic robot mighty[27] which is shown in Fig.4. The total mass of the model is lumped at COM, for simplicity. The sideward stance width between the both feet was 0.1[m] ($r = 0.05$[m]), the height of COM was $z = 0.26$[m], $q_y = 1$ and $k = 1$. Each foot is represented by a rectangle with the forward length 0.055[m], the backward length 0.04[m], the innerward length 0.035[m] and the outerward length 0.035[m]. While each foot turned to the swinging foot, a second-order lag controller to track smoothly the desired landing foot position defined by Eq.(14) was validated as

$$\ddot{x}_* = K({}^{d}x_S - x_*) - C\dot{x}_*, \tag{15}$$

where $*$ is for L or R, and x_L and x_R mean the position along x axis of the left foot and the right foot, respectively. $K = 3000$ and $C = 50$ were adopted in the simulation. As already explained in the previous section, the up-and-down motion of the swinging foot was controlled so as to synchronize to the lateral oscillation of ZMP to guarantee the condition (3). The saturation rules in Eqs.(5) and (11) are substituted for the proximity query from the desired ZMP to the supporting region for a generalized form as

$$\boldsymbol{p}_Z = \arg \min_{\bar{\boldsymbol{p}}_Z \in \boldsymbol{S}} \|\bar{\boldsymbol{p}}_Z - \tilde{\boldsymbol{p}}_Z\|. \tag{16}$$

The differential equations were numerically solved by fourth-order Runge-Kutta's method with the quantized interval 0.01[s].

We developed a simulation program which enables interactive modifications of each control parameter via manipulations of sliders on the window. Those modifications are immediately reflected to the dynamics computation. Fig.5 shows its screenshot. The blue zone (for readers with color) in the phase portrait at the bottom means the stabilizable region, and the orange zone (again, for readers with color) means the supporting region where the robot motion is dynamically feasible if ZMP (the green point for readers with color) exists within this region.

The initial condition was set for $(x, \dot{x}, y, \dot{y}) = (0.0409, 0, 0, 0)$, $q_x = 0.5$ and $\rho = 0$. During the simulation, ${}^{d}v_x$ was modified within the range of $-3.0 \sim 3.0$[m/s] at random via the slider, except for the last phase of the simulation in which ${}^{d}v_x$ was set for 0[m/s] in order to return to the standing stabilization. Fig.6 shows snapshots of the movie of the simulation. The history of the referential velocity of COM, the actual velocity of COM, the position of COM, the position of ZMP, the positions of feet and the supporting region are plotted in Fig.7.

Fig.7(a) shows that the velocity of COM along x axis tracks the referential value. The position of COM, the position of ZMP, the desired position of the landing, the positions of the both feet and the supporting region along x axis are plotted in Fig.7(b). The position of COM, the position of ZMP and the supporting region along y axis are shown in Fig.7(c). One can observe in the graphs that ZMP always lies within the supporting region but is frequently stuck at the edge of the region during the motion, particularly the backward walking motion for a negative referential velocity, and accordingly COM becomes uncontrollable at the corresponding moments. At the moment of landing, COM comes back into the controllable region so that the velocity of COM converges to the referential value, while discontinuous jumps of ZMP yield nonsmooth change of the velocity.

Despite the referential velocity of COM was changed completely at random, the manipulation of ZMP, the deformation of the supporting region and the foot steps were consistently done in both x-y directions, and a continual walk including starting, turnings and stopping was achieved.

Another simulation to see a flexible behavior against perturbations was conducted on the same program with the same

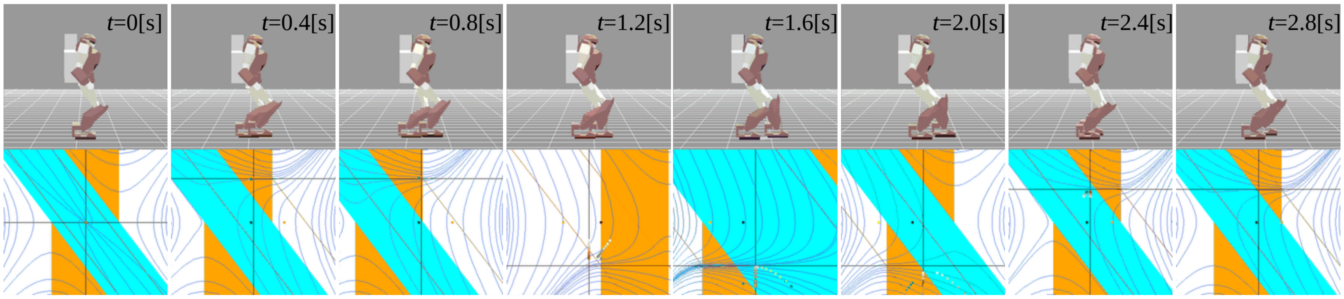


Fig. 6. Snapshots of the movie of the simulation of the velocity following control

initial condition $(x, \dot{x}, y, \dot{y}) = (0.0409, 0, 0, 0)$, $q_x = 0.5$ and $\rho = 0$. ${}^d v_x$ was also modified within $-3.0 \sim 3.0$[m/s] at random, except for the last phase in which ${}^d v_x$ was set for 0[m/s] as well as the previous simulation. During a forward walking, external forces along x axis were applied twice as perturbations. The history of the referential velocity of COM, the actual velocity of COM, the position of COM, the position of ZMP, the positions of feet, the supporting region and the applied external force are plotted Fig.8. As shown in Fig.8(c), a backward external force was exerted from about 0.09[s] to 0.16[s], and then, a forward external force was exerted from about 0.24[s] to 0.28[s]. One can see that the motion of COM is perturbed during the terms in Fig.8(a) and (b). Since the behavior of the robot was not constrained by any time-dependent trajectory, the motion was able to be easily modified from that without perturbations, and the robot succeeded to keep walking.

V. Conclusion

A novel biped controller which is free from time-slaved trajectories was developed based on the dynamics morphing. The velocity following walking control in longitudinal direction was achieved even with respect to arbitrary referential velocities which is given at random timings, and also under certain magnitudes of unexpected perturbations exerted at random. It enables seamless transitions from/to the standing stabilization simply by modifying one of the system poles to be zero. The success of continued walking also stands on some techniques including a foot control maneuver to guarantee the standing stabilizability immediately after landing, and automatic update of the referential position of COM.

The maximum referential velocity which the robot could follow achieved in the simulations was about 0.3[m/s]. Since the height of the robot is 0.54[m], it corresponds to 1.0[m/s]=3.6[km/h] for a 1.8[m] tall person. One may feel it slower than a regular person's pace. The author found that the limitation is not due to the proposed control method but rather due to the maximum stride. In all the simulations conducted in the previous section, the robot kept foot-flat when contacting with the ground for simplicity, so that the feasible stride was not large. In order to improve the performance, a dextrous foot mechanism with movable toes and a skillful pedipulation utilizing them is required. It is another issue to be solved in the future.

For the turning control to change the walking direction, the controller should be reformulated with respect to the rotating frame attached to the robot body, and the interference between the longitudinal and lateral motions should be taken into account. It will be reported in the next chance.

To the best of the author's knowledge, there hasn't been proposed a biped walking controller which is compatible with the standing stabilization without relying on the time-dependent trajectories so far, despite the biped robot control has been studied over 40 years. He believes that the proposed control scheme will suggest a new methodology to develop reliable biped robots.

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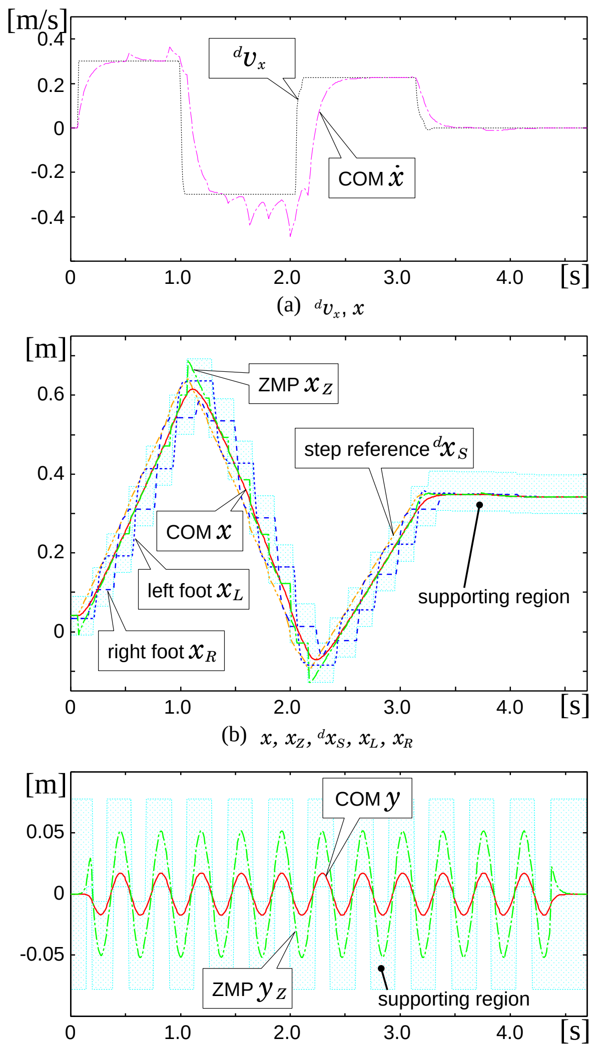
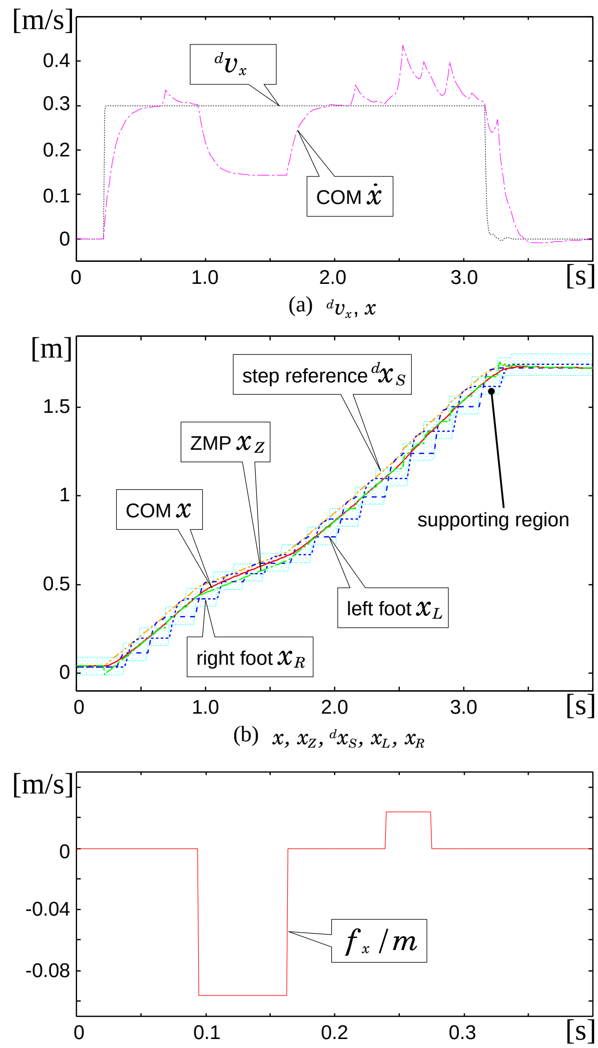


(a) ${}^{d}v_x$, x

(b) x, x_Z, ${}^{d}x_S$, x_L, x_R

(c) y, y_Z

Fig. 7. Results of the simulation of the velocity following control



(a) ${}^{d}v_x$, x

(b) x, x_Z, ${}^{d}x_S$, x_L, x_R

(c) y, y_Z

Fig. 8. Results of the simulation of the velocity following control

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