2D Omnidirectional Navigation of a Biped Robot
Based on an Egocentric Orbit Following

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Abstract—This paper addresses an omnidirectional locomotion control to make a biped robot walk back and forth, turn to any direction, move sideways and carry out them seamlessly. This is achieved by integrating smooth-path-tracking and lateral walking controllers that have been proposed by the authors, which is not straightforward because the path tracking is designed to converge to a referential path while the lateral walking aims to go away from the path. It can be avoided by extending the path-tracking controller to consecutively redraw the orbit that passes the middle of the feet. The proposed controller is designed completely in the robot-centric frame, namely, all the references and the manipulated variables are represented from the robot’s viewpoint so that the operator can issue the referential values in the first-person view. Additionally, as the robot is ruled by neither a plan of footsteps nor time-dependent trajectories, it can flexibly respond to various perturbations. The validity of the proposed controller is investigated through computer simulations and an application to navigation of the robot is also shown.

I. INTRODUCTION

Omnidirectional locomotion is essential for biped robots to travel in an unstructured environment. When teleoperating the robot interactively, responsibility to follow changes of motion commands and robustness to unexpected events are required. Additionally, a robot-centric controller enables the operator to navigate it only from local information, which is favorable for visually-guided navigation.

Biped locomotion has been successfully achieved by planning a sequence of footsteps followed by a detailed referential whole-body trajectory from the beginning to the end [1]–[5]. This is effective only in situations where there is less constraint on time to plan and the surrounding of the robot is almost invariant during the operation. An extension of this approach is an online planning of foot placements and whole-body coordination [6]–[12], which increases the capability to deal with disturbances and changes of motion commands. However, they still lack responsibility since replanning of the trajectory is restricted on a step to step basis or update cycle of the controller reaches a ceiling due to its computational cost to solve some optimization problem.

An empirically-designed omnidirectional walking controller has been proposed [13]–[15], which can accept motion commands in a robot-centric scope at any timing. They have been put into practical uses, and recovery from emergent situations, for instance due to external forces, is achieved in an event-driven way. However, one has to prepare a remedy for each individual irregularity. Moreover, it often requires learning process to optimize parameters.

The goal of this paper is to develop an omnidirectional control for biped robots by combining a smooth-path-tracking control and a lateral walking control which have been proposed by the authors [16], [17]. The former makes the robot follow a given referential path that is defined by a local curvature at the current position while the latter leaves the robot from the path. To resolve the conflict, the path tracking control is extended to adaptively redraw the local orbit to follow based only on egocentric information instead of the given path. This extension compensates for the deviation from the path by the lateral walking. The proposed method does not need to plan a sequence of footsteps, time-dependent trajectories and event-driven behaviors, and all the references and manipulated variables are represented in the egocentric frame of reference, namely, they are represented with respect the moving frame attached to the center of mass (COM) of the robot. The desired positions of the zero moment point (ZMP) and the both feet can be instantaneously determined from the curvature of the path and commanded velocity. The advantage of the proposed method include responsibility to updates of references, robustness to external disturbances and maneuverability for remote operation.

The overview of our control scheme is shown in Fig. 1. The references include curvature of the referential orbit and velocities along the tangential and orthogonal directions to the orbit. The calculation of the egocentric orbit to follow is the main contribution of this work, which will be described in section IV. The COM controller computes the desired ZMP position and obtains the desired COM position based on the COM-ZMP model [18] as described in section II. The feasibility of ZMP is checked in this process. The objective
of the foot controller is to coordinate the foot and COM motions. The lateral walking control is implemented as a part of the foot control, which will be detailed in section III. The joint displacements of the whole body are obtained by solving the inverse kinematics from the desired COM and foot positions. As stressed below, all the references and manipulation variables are represented in the robot-centric frame.

II. SMOOTH-PATH-TRACKING BIPED CONTROL [17]

A simplified model of a biped robot depicted as Fig. 2(a) is considered, where the total mass of the robot is concentrated at COM. Let us assign a coordinate system $\Sigma_W$ in which $z$ axis is along with the upward direction and $x$ and $y$ axes are determined by the right-hand rule. Assuming that the height is constant, the moment about COM is small enough to be neglected and the ground reaction forces are distributed on a horizontal plane, the following equation is obtained:

$$\dot{x} = \zeta^2 (x - x_Z)$$
$$\dot{y} = \zeta^2 (y - y_Z),$$

where positions of COM and ZMP [19] are denoted by $p = [x \ y \ z]^T$ and $p_Z = [x_Z \ y_Z \ z_Z]^T,$ respectively, $\zeta \equiv \sqrt{g/z}$ and $g = 9.81$[m/s$^2$] is the gravitational acceleration. Eqs. (1)(2) indicate that COM can be controlled through ZMP manipulation [18], $p_Z$ is constrained to lie within the supporting region, namely, the convex hull of all contact points between the feet and the ground, which is represented as follows:

$$p_Z \in S.$$ 

Notice that it changes discontinuously during motion.

Suppose that the robot walks along an arc centered at the origin of $\Sigma_W$. The COM $[x \ y]^T$ and ZMP $[x_Z \ y_Z]^T$ with respect to $\Sigma_W$, which are projected on the plane, can be described with respect to the polar coordinates as follows:

$$[x] = r [\cos \theta \sin \theta], \quad [x_Z] = r_Z [\cos \theta_Z \sin \theta_Z],$$

where $(r, \theta)$ and $(r_Z, \theta_Z)$ are radii and angles to COM and ZMP, respectively. With the relationship, Eqs. (1)(2) reduced to the following (detailed in [17]):

$$2\dot{\theta} + r\dot{\theta} = \zeta^2 r_Z \sin(\theta - \theta_Z)$$
$$-r + \dot{r} = -\zeta^2 (r - r_Z \cos(\theta - \theta_Z)).$$

Let us consider another coordinate system $\Sigma_M$ whose origin is located just below the position of COM, in which $u$ and $w$ axes are oriented to the forward and leftward directions of the robot, respectively (Fig. 2(a)). For ZMP position, the following relation holds:

$$\begin{bmatrix}
 u_Z \\
 w_Z
\end{bmatrix} =
\begin{bmatrix}
 -r_Z \sin(\theta - \theta_Z) \\
 r - r_Z \cos(\theta - \theta_Z)
\end{bmatrix},$$

where $[u_Z \ w_Z]^T$ denotes ZMP position with respect to $\Sigma_M$. Substituting Eq. (7) into Eqs. (5)(6) gives the equation of motion with respect to $\Sigma_M$.

$$2\dot{\theta} + r\dot{\theta} = -\zeta^2 u_Z,$$
$$-r + \dot{r} = -\zeta^2 w_Z.$$ The terms $r\dot{\theta}^2$ and $2\dot{\theta}\dot{r}$ correspond to accelerations due to the centrifugal and Coriolis forces, respectively.

Let us consider $u_Z$ and $w_Z$ as control inputs. A curved walking control is proposed as follows [17]:

$$u_Z = -q_u \eta \Delta U + \frac{q_u + 1}{\zeta} (v_u - \eta^d v_u) + \frac{2\kappa v_u v_w}{\zeta^2 \eta}$$
$$w_Z = -q_w \Delta U + \frac{q_w + 1}{\zeta} \gamma(d) v_w - \frac{\kappa v_u^2}{\zeta^2 \eta},$$

where

$$\eta \equiv 1 + \kappa \Delta w,$$
$$d \equiv \sqrt{\Delta w^2 + \frac{v_w^2}{\xi^2 v_w}},$$
$$\gamma(d) \equiv 1 - \rho \exp\left(1 - \frac{(q_w + 1)^2 d^2}{d_0^2}\right).$$

Fig. 2(b) shows variables used in the controller. $^d r$ is the radius of the circular path. $^d v_u$ is the referential velocity along the tangential to the path. $\kappa$ is the local curvature of the path, namely, $\kappa = 1/\xi^d r$. $\Delta U$ is the arc length to the referential position along the path. $\Delta w$ is the displacement between COM and the path along the radial direction. $v_u$ and $v_w$ are the velocities of COM with respect to $\Sigma_M$. It should be noted that the references and state variables in Eqs. (10)(11) are described with respect to $\Sigma_M$.

Various motions can be achieved by setting the control parameters, $q_u$, $q_w$, $\kappa$, $\rho$ and $d$ to appropriate values. When setting $^d v_u$ and $\rho$ to 0, the controller becomes the stability-maximized COM-ZMP regulator [20]. When setting $\rho$ to 1, the following stable limit cycle appears along $w$ axis:

$$\Delta w^2 + \frac{v_w^2}{\zeta^2 v_w} = \frac{d^2}{(q_w + 1)^2},$$

which produces lateral oscillation for stepping [21], $q_w$ and $d$ determine the period $T = \frac{2\pi}{\zeta \sqrt{q_w}}$ and amplitude $R = \frac{d}{q_w + 1}$. 

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of oscillation. \( k \) determines the degree of convergence to the limit cycle. When setting \( q_u \) for 0, it works as a velocity-follower which converges \( v_u \) to \( \dot{q}_w \) for curved walking. Straight walking is achieved when \( v = 0 \).

In order for the robot to keep walking, the supporting region has to be deformed so as to satisfy Eq. (3) consistently with COM movement. Let us consider the transition between double- and single-support phases. Owing to Eq. (3), the right foot in Fig. 3(a) can be lifted up while ZMP lies in the left sole. Hence, we have to estimate when ZMP comes into contact with the supporting foot. To avoid the collision, a constraint to preclude the desired landing position between two concentric arcs that pass the feet as shown in Fig. 4 is imposed, and it is represented as follows:

\[
\begin{bmatrix}
\dot{u}_\text{KF} \\
\dot{w}_\text{KF}
\end{bmatrix} = \begin{bmatrix}
\frac{u_{\text{SP}} - q_u}{\zeta} + \frac{\Delta u}{\zeta^2 (1 + \kappa \Delta w)} \\
\frac{w_{\text{SP}} - q_w}{\zeta} + \frac{\Delta w}{\zeta^2 (1 + \kappa \Delta w)}
\end{bmatrix}
\]

where

\[
\begin{align*}
\tan \varphi_{\text{KF}} &= \frac{k u_{\text{SP}}}{1 + \kappa \Delta w - \kappa w_{\text{SP}}} \\
\Delta u_{\text{SP}} &= \text{sp}_\text{KF} \tan \frac{\varphi_{\text{KF}}}{2} + \Delta w - w_{\text{SP}} \\
\Delta r_{\text{KF}} &= \delta - \Delta w_{\text{SP}}.
\end{align*}
\]

\( \varphi_{\text{KF}}, \Delta w_{\text{SP}} \) and \( \Delta r_{\text{KF}} \) are indicated in Fig. 4, which are obtained from geometric relationships shown in Appendix.

### III. IMPROVEMENT OF LATERAL LOCOMOTION CONTROL

#### A. Dynamics of lateral biped locomotion

Lateral locomotion has a complex nature from the viewpoint of the dynamical system. Since the pair of legs are aligned in parallel to the moving direction, the robot has to accelerate and decelerate COM cyclically during the transportation to avoid the collision. This motion can be visualized on the phase space as Fig. 5(a). As can be seen from it, the trajectory of COM intersects with itself, which means that the motion cannot be achieved by a time-invariant controller and it is required to change the dynamical structure during motion.

In the previous work [16], as shown in Fig. 5(b), COM is accelerated by a velocity-follower when the robot is supported by the opposite foot with the moving direction, on the other hand, it is decelerated by a half-cut self-excited oscillator which also works to get the opposite foot close to the supporting foot when the robot is supported by the other foot. However, as the foot-lifting control was designed on the basis of the limit cycle, the foot height could not be defined during acceleration phase. Although a workaround for the problem was shown in [16], it produced unstable response at the moment of the controller switching.

#### B. New control strategy for lateral locomotion

The velocity-follower caused a mismatch with the phase-driven control in the previous method. An improved way achieves the cyclic acceleration and deceleration of COM by expanding and shrinking the amplitude of the limit cycle in accordance with the contact state of the supporting foot as shown in Fig. 5(c).
Let us consider the robot behavior in the acceleration and deceleration phases. In the acceleration phase, the legs are set to open gradually as $\phi$ evolves. In the deceleration phase, the legs move exactly in the opposite manner. As the amplitude of the limit cycle is determined by $\phi$, we can accomplish by redefining it as follows:

$$
\bar{d} = \begin{cases} 
\bar{d}_0 + \frac{\pi}{\xi} \sqrt{\eta} \phi [d_u'/w'] & \text{(acceleration phase)} \\
\bar{d}_0 + (\bar{d}_0 - \bar{d})\phi & \text{(deceleration phase)}
\end{cases}
$$

(27)

where $d$ and $\bar{d}_0$ are the actual and nominal width of the feet, respectively.

IV. CALCULATION OF REFERENTIAL ORBIT TO FOLLOW IN EGOCENTRIC FRAME

The integration of the smooth-path-tracking controller and the lateral walking controller is not straightforward because the former is designed to converge to a referential curved path while the latter aims to go away from it. This is attributed to the variable width of the feet. In curved walking, the center of turning (the point $O_w$ in Fig. 6) is invariant because it can be assumed that the width of the feet is constant. When the robot tries the curved and lateral walking simultaneously, the changing width of the feet due to Eq. (27) violates the assumption.

To resolve the issue, it is extended to calculate the referential orbit to pass the middle of the feet instead of the constant center of turning. With this extension, we need only to compute the center of turning (the point $O'_w$) from local information. Practically, since the curvature $\kappa = 1/d_r$ is given, it is sufficient if we get the point $[d_u' \quad d_w']$ with respect to $\Sigma_M$.

Let us denote the desired COM position and the referential position after $\Delta t$ second as $[u' \quad w']$ and $[d_u' \quad d_w']$, respectively, as shown in Fig. 6. The orbit which passes $[d_u' \quad d_w']$ is called the referential orbit. $[r_u' \quad r_w']$ is the referential position if the width of the feet were unchanged, and the orbit which passes the point is called the auxiliary orbit. The distance from $O_W$ to the referential position after $\Delta t$ second is $d_w'$. The distance from the desired COM position to the referential position after $\Delta t$ second is $\Delta u'$, and to the referential position if the width of the feet were unchanged is $\Delta \bar{u}'$. The following relationship is hold.

$$
\Delta w' = d_r + \Delta \bar{u}' - d_r'
$$

(28)

From the condition that the center of oscillation is located at the middle of the feet,

$$
d_r' = \frac{1}{2}(r_{KF} + r_{PF}).
$$

(29)

We can obtain the following relations (see Appendix).

$$
tan \varphi' = \frac{\kappa u'}{\eta - \kappa w'}
$$

(30)

$$
\Delta \bar{w}' = u' tan \varphi' + \Delta w - w'.
$$

(31)

Similarly to the above, we can get the following:

$$
r_{KF} = d_r + \Delta w_{KF}
$$

(32)

$$
tan \varphi_{KF} = \frac{\kappa u_{KF}}{\eta - \kappa w_{KF}}
$$

(33)

$$
\Delta w_{KF} = u_{KF} tan \varphi_{KF} + \Delta w - w_{KF}.
$$

(34)

$r_{PF}$, $tan \varphi_{PF}$ and $\Delta w_{PF}$ are also obtained in the same way.

From Eqs. (28)(29)(32) and $r_{PF}$,

$$
\Delta w' = \Delta \bar{w}' - \frac{1}{2}(\Delta w_{KF} + \Delta w_{PF}).
$$

(35)

Therefore, the referential position of COM after $\Delta t$ second $[d_u' \quad d_w']$ can be obtained as

$$
\begin{bmatrix} d_u' \\ d_w' \end{bmatrix} = \begin{bmatrix} u' \\ w' \end{bmatrix} + \Delta w' \begin{bmatrix} -sin \varphi' \\ cos \varphi' \end{bmatrix}.
$$

(36)

V. SIMULATIONS AND DISCUSSION

A. Simulations

Simulations to validate the proposed controller were executed. An anthropomorphic robot “mighty” [24] was supposed, with the total mass concentrated at COM for simplicity in the model. The control parameters were set for $z = 0.26[m]$, $q_u = 0.5$, $k = 1$ and $h_{max} = 0.02[m]$. 

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Simulation A was conducted to make sure that motion commands can be changed during motion. Motion commands, $d_vu$, $d_vw$ and $\kappa$, were modified at random during the simulation. Fig. 7 shows a sequence of the footprints and trajectories of actual COM and ZMP. As shown in Fig. 7, the robot could walk freely and stop when motion commands were updated without falling.

Simulation B was conducted to make sure that the robot can stabilize against external forces. Random magnitudes and directions of forces were applied to COM every 7 seconds. Fig. 8(a) shows trajectories of actual COM and ZMP, footprints and forces indicated by blue arrows. As it can be observed, the robot could respond to external disturbances by stepping out the foot. Fig. 8(b) shows a magnified view of footprints when the robot is subjected to the first force. Dotted lines are footprints if the force were not applied. Compared with them, the robot could respond to unknown disturbances by stepping. Fig. 8(c) shows the height of the feet around when the first force is applied. The timings to lift off and touch on the ground were flexibly modified.

Simulation C was conducted to evaluate the maneuverability in visually-guided navigation. The proposed controller requires $d_vu$, $d_vw$ and $\kappa$ as motion references, which can be provided from first-person perspective of the robot. To make use of this characteristic, we implemented a simple application shown in 9(b). The task was to navigate the robot to a goal in an environment surrounded by desks, chairs and bookshelves. Motion reference were correspondent with joystick’s inputs. The task could be achieved as shown in Fig. 9(c).

B. Discussion

To apply the proposed controller to a real robot, we have some issues. First, in our control scheme, the actual ZMP is assumed to follow the desired ZMP. However, as ZMP cannot be moved directly, we have to manipulate it through the whole body motion. Though how to compensate the errors is out of the scope of this paper, introducing an external force observer could be considered. Second, to feedback the current state of the robot an estimation of COM is required. The estimation of COM is generally a difficult problem, while a slight amount of errors can be handled as well as external forces. It should be evaluated how much amount of errors can be allowed. Third, the controller assumes the ground is open and flat. It does not suit to environments where footholds are limited. However, on uneven terrains which have small bumps like gravelly road, the fluctuation can also be handled as external disturbances. We should investigate the stability on those terrains.

VI. Conclusion

A novel biped controller which enables a robot to achieve omnidirectional locomotion was proposed. The controller was developed by combining two controllers for curved and lateral walking, which have been proposed by the authors. To integrate them, the lateral locomotion controller was improved to avoid controller switching that produced an
Fig. 10: Geometric relationship between a point and origins of the frames.

unstable response, and the curved walking was improved to handle the variable width of the feet. The validity of our controller was examined through computer simulations and an application to remote operation of humanoids was shown.

APPENDIX

Fig. 10 shows a geometric relationship between a point w.r.t $\Sigma_M$ and the origins of the two coordinate systems. Let us represent $\Delta w_p$ with $\Delta w$, $\kappa$, $U$ and $W$.

The angle $\alpha$ satisfies the following:

$$\sin \alpha = \frac{U}{\sqrt{U^2 + W^2}}, \quad \cos \alpha = \frac{W}{\sqrt{U^2 + W^2}}. \quad (37)$$

Using the law of sines for $\triangle P O W O_M$,

$$\frac{U}{\sin \varphi_p} = \frac{d_r + \Delta w}{\sin(\varphi_p + \alpha)}. \quad (38)$$

We can get the following equation from Eq. (37),

$$\sin(\varphi_p + \alpha) = \frac{1}{\sqrt{U^2 + W^2}} (U \cos \varphi_p + W \sin \varphi_p). \quad (39)$$

Eqs. (38)(39) provide the relationship between $\varphi_p$ and $\kappa$,

$$\tan \varphi_p = \frac{U}{d_r + \Delta w - W} = \frac{\kappa U}{1 + \kappa \Delta w - \kappa W}, \quad (40)$$

$$\frac{1}{\kappa} = \frac{U}{\tan \varphi_p} - (\Delta w - W). \quad (41)$$

We get $\Delta w_p$

$$U = (d_r + \Delta w_p) \sin \varphi_p$$

$$\Leftrightarrow \Delta w_p = \frac{U}{\sin \varphi_p} - \frac{1}{\kappa} = \frac{U}{\sin \varphi_p} - \frac{U}{\tan \varphi_p} + (\Delta w - W)$$

$$= U \tan \varphi_p - \frac{U}{2} + \Delta w - W. \quad (42)$$

REFERENCES


